MSc Math April 2015



Seat Number

* 3 0 2 7 *

# MT-401
# Integral Equations and Transforms
## (401)

**P. Pages : 3**

**Time : Three Hours** **Max. Marks : 80**

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Instructions to Candidates :

1. Do not write anything on question paper except Seat No.
2. Graph or diagram should be drawn with the black ink pen being used for writing paper or black HB pencil.
3. Students should note, no supplement will be provided.
4. Attempt **any five** questions of the following.
5. All questions carry equal marks.
6. Figure to the right indicates full marks.

**1.** a) Convert the differential equation $y''(x)-3y'(x)+2y(x)=5\sin x,\ y(0)=1,\ y'(0)=-2$ into an integral equation. **5**

b) Solve the Abel's integral equation $\int_0^x \frac{y(t)}{\sqrt{x-t}}dt=1+2x-x^2$. **6**

c) Show that the function $y|x|=(1+x^2)^{-3/2}$ is a solution of the voltervaintegral equation : **5**

$$y(x)=\frac{1}{1+x^2}-\int_0^x \frac{t}{1+x^2}y(t)dt$$

**2.** a) State and prove linearity property of Laplace transform. **5**

b) Find the Laplace transform of $\frac{\sin at}{t}$ Does the transform of $\frac{\cos at}{t}$ exist ? **6**

c) Show that $\int_0^\infty t\, e^{-3t}\sin t\, dt=\frac{3}{50}$ **5**

3. a) Let F(t) be a periodic function with period w so that F(t+w) = F(t). Then prove that 8

$$L\{F(t)\} = \int_0^w \frac{e^{-st}F(t)\,dt}{1-e^{-sw}}$$

b) Find the inverse Laplace transform of the following functions ?

i) $f(s) = \frac{e^{-1/s}}{s}$ 4

ii) $f(s) = \frac{s}{(s^2+a^2)^2}$ 4

4. a) By the method of residues, find 8

$$L^{-1}\left\{\frac{1}{(S-1)(S+2)(S-3)}\right\}$$

b) Show that $\phi(x) = \frac{1}{\pi\sqrt{x}}$ is a solution of the integral equation $\int_0^x \frac{\phi(u)\,du}{\sqrt{x-u}} = 1$, by using convolution property of Laplace transform. 4

c) Using Laplace transform, solve the differential equation $y'' + y = \cos x,\ y(0) = 0 = y'(0)$. 4

5. a) By providing an example which establish the relation between Fourier and Laplace transforms. 5

b) State and prove the change of scale property for Fourier transform. 7

c) Find Fourier sine transform of $F(x) = \frac{1}{x}$. 4

6. a) Find the finite sine transform of the following :

i) $e^{ax}$ 3

ii) $\sin ax$ 4

iii) $x^3$ 5

b) Whether the following functions are of exponential order or not as $t \to \infty$ ? **4**

i) $F(t) = t^n$.

ii) $F(t) = e^{t^2}$

**7.** a) Apply Laplace transform to solve the partial differential equation $\frac{\partial y}{\partial x} - \frac{\partial y}{\partial t} = 1 - e^{-t}, 0 < x < 1, t > 0$ with y(x, 0) = x. **8**

b) Solve the integral equation by Fourier transforms for F(x) ? **8**

$$\int_0^\infty F(x)\sin(xt)dx = \begin{cases} 1 \ , & 0 \le t < 1 \\ 2 & 1 \le t < 2 \\ 0 & t \ge 2 \end{cases}$$

**8.** a) Show that the geodesics on a sphere of radius a are its great circles. **8**

b) If x does not occur explicitly in F, then prove that $F_{y'}y' - F = \text{constant}$. **4**

c) Find the extremals of the functional $I[y(x)] = \int_1^3 y(2x - y)dx$. **4**

Which satisfies the boundary conditions $y(1) = 0 = y(0)$.

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Seat Number

*3028*

# MT-101
# Advanced Calculus
## (New) (111101)

**P. Pages : 3**

**Time : Three Hours**

**Max. Marks : 80**

Instructions to Candidates :
1. Do not write anything on question paper except Seat No.
2. Graph or diagram should be drawn with the black ink pen being used for writing paper or black HB pencil.
3. Students should note, no supplement will be provided.
4. Attempt **any five** questions.
5. Figures to the right indicate full marks.

1. a) Show that $C = 2^{N_0}$ where C is the cardinal number of the set of all real numbers $\mathbb{R}$ and $N_0$ is the cardinal number of the set of rational numbers. **6**

   b) Define Cantor's ternary set and show that it is a closed set, an uncountable set of measure zero. **8**

   c) Write note on the Lebesgue function. **2**

2. a) Let $E \subseteq R$. Prove that following three conditions are equivalent : **8**
   i) E is measurable.
   ii) For any $\in > 0$, there is an open set $O \supseteq E$ such that $m^*(O - E) < \in$.
   iii) There is a $G_\delta$-set G such that $G \supseteq E$ and $m^*(G - E) = 0$.

   b) Prove that there exists a non-measurable set in $\mathbb{R}$. **8**

3. a) Suppose f and g are real valued functions defined on all of $\mathbb{R}$, f is measurable and g is continuous. Is the composition fog necessarily measurable ? Justify. **7**

   b) Construct a function f such that |f| is measurable function but f is not. **3**

   c) Let f be an extended real valued function with f(x+y) = f(x) + f(y) for all x, y. Show that if f is measurable, then f(x) = xf(1) for each x. **6**

4. a) Let $\{f_n\}$ be a sequence of measurable functions on the same measurable set. Prove that : 8
   i) $\sup_{1\le i\le n} f_i$ is measurable for each n.
   ii) $\inf_{1\le i\le n} f_i$ is measurable for each n.
   iii) $\sup f_n$ is measurable
   iv) $\inf f_n$ is measurable
   v) $\lim\sup f_n$ is measurable
   vi) $\lim\inf f_n$ is measurable.

   b) let f and g be non negative measurable functions. Prove that $\int(f+g)dx = \int fdx + \int gdx$ 3

   c) State and prove Lebesgue's Dominated convergence theorem. 5

5. a) Show that every measurable function f there corresponds a Borel-measurable function of such that f = g a.e. 6

   b) Let f and g be integrable functions and $a \in \mathbb{R}$. Prove that 10
   i) af is integrable, and $\int afdx = a\int fdx$
   ii) f+g is integrable, and $\int(f+g)dx = \int fdx + \int gdx$
   iii) If f = 0 a.e. , then $\int fdx = 0$
   iv) If $f \le g$ a.e. then $\int fdx \le \int gdx$
   v) If A and B are disjoint measurable sets then
   $\int_A fdx + \int_B fdx = \int_{A\cup B} fdx$

6. a) If f is Riemann integrable and bounded over the finite interval [a, b] then show that f is integrable and $R\int_a^b fdx = \int_a^b fdx$. 8

   Give an example of a Lebesgue integrable function on an internal which is not Riemann integrable.

   b) Define a function of bounded variation. Prove that $f \in Bv[a, b]$ if and only if f is the difference of two finite valued monotone increasing functions on [a, b] where a and b are finite. 8

7. a) Let [a, b] be finite interval and let $f \in L[a, b]$ with indefinite integral F. Show that $F' = f$ a.e. in [a, b]. 8

b) Define g on [0, 1] by $g(\theta) = 0, g(x) = x \sin\left(\frac{\pi}{x}\right)$ for x > 0. Show that g is continuous but that $g \notin B \vee [0, 1]$. **5**

c) Give an example where $D^{+}(f+g) \neq D^{+}f + D^{+}g$. **3**

**8.** a) State and prove Vitali's covering theorem. **8**

b) If f and g are absolutely continuous on the finite interval [a, b] then show that fg is absolutely continuous on [a, b]. **4**

c) Give an example of a function which is continuous but not absolutely continuous. **4**

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Seat Number

* 3 0 2 9 *

# MT-101
# Real Analysis
## (Old) (101)

**P. Pages : 3**

**Time : Three Hours** **Max. Marks : 80**

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Instructions to Candidates :
1. Do not write anything on question paper except Seat No.
2. Graph or diagram should be drawn with the black ink pen being used for writing paper or black HB pencil.
3. Students should note, no supplement will be provided.
4. Attempt **any five** questions.
5. Figures to the right indicate full marks.

**1.** a) Prove that a countable union of a countable set is countable. **6**

b) Show that the set P(N) of all subsets of N is uncountable. **6**

c) Prove that $C = 2^{No}$, where C is cardinal number of R and No is cardinal number of 2. **4**

**2.** a) Show that an infinite set has denumerable subset. **5**

b) Prove that [0,1] and (0,1) are equivalent set. **5**

c) Let {In} be finite set of intervals covering the rationals in [0,1]. Show that $\sum l(Ih) \geq 1$. **6**

**3.** a) Prove that outer measure of an interval equals its length. **8**

b) Show that if $f \in$ M and m(F∆G)=0 then G is measurable. **4**

c) Prove that the class M is a algebra. **4**

**4.** a) Let C be any real number and let f and g be real valued measurable functions defined on E. Then prove that f+c, cf, f+g are also measurable. **8**

b) Show that $f \leq$ ess sup f a.e. 4

c) Let f be defined on [0,1] by f (0) = 0, $f(x)=x \sin\left(\frac{1}{x}\right)$ for x > o. Find measure of the set $\{x : f(x) \geq 0\}$. 4

5. a) Let f and g be non negative measurable functions. Then prove that. 6

   i) If A is measurable set and $f \leq g$ on A then $\int_A f dx \leq \int_A g dx$

   ii) If $a \geq 0$, then $\int afdx = \int fdx$

   iii) If A and B are measurable sets and $A \supseteq B$ then $\int_A f dx \geq \int_B f dx$

b) State and prove Fatou's lemma. 6

c) Show that $\int_1^\infty \frac{dx}{x} = \infty$ 4

6. a) Let f and g be integrable functions. Then prove that. 6

   i) af is integrable and $\int afdx = a\int f dx$

   ii) f+g is integrable and $\int (f+g)dx = \int fdx + \int g dx$.

   iii) If f = O a.e. then $\int f dx = o$.

b) Show that $\int_0^1 \frac{x^{1/3}}{1-x} \log\left(\frac{1}{x}\right) dx = g\sum_{h=1}^{\infty} \frac{1}{(3h+1)^2}$. 6

c) If f is continuous on the finite interval [a,b], then prove that f is integrable and f (x) = $\int_a^x f(t)dt$ (a<x <b) is differentiable function such that $f^1(x) = f(x)$. 4

7. a) Let f be a bounded function defined on the finite interval [a,b] then prove that f is Riemann integrable over [a,b] iff it is continuous a.e. **8**

b) Let f $\epsilon$ BV [a,b] then prove that f (b) – f (a) = P-N and T=P+N all variations being on the finite interval [a,b]. **8**

8. a) If f is finite valued monotone increasing function defined on the finite interval [a,b] then prove that $F^1$ is measurable and **8**

$$\int_a^b f^1 dx \leq f(b) - f(a).$$

b) Let [a,b] be a finite interval let f $\epsilon$ L (a,b) with indefinite integral F then prove that $F^1 = f$ a.e. in [a,b]. **8**

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Seat Number

*3030*

# MT - 301
# Functional Analysis
# (301)

P. Pages : 2

**Time : Three Hours** **Max. Marks : 80**

Instructions to Candidates :
1. Do not write anything on question paper except Seat No.
2. Graph or diagram should be drawn with the black ink pen being used for writing paper or black HB pencil.
3. Students should note, no supplement will be provided.
4. Attempt **any five** questions.
5. Figures to the right indicate full marks.

**1.** a) Define : **3**
   i) Normed linear space.
   ii) Banach space.
   iii) Hilbert space.

   b) Show that the linear space $\mathbb{R}^n$ of real numbers are Banach space under the norm $\|x\| = \left( \sum_{i=1}^{n} |x_i|^2 \right)^{1/2}$ **8**

   c) If T is a continuous linear transformation of a normed linear space N into a normed linear space N', and if M is it's null space then show that T induces a natural linear transformation T' of N/M into N' and $\|T'\| = \|T\|$. **5**

**2.** a) State and prove Hahn-Banach Theorem. **8**

   b) If N is a finite dimensional normed linear space of dimension n then show that $N^*$ also has dimension n and N is reflexive. **8**

**3.** a) State and prove the Banach Steinhaus theorem. **8**

   b) If B is a Banach space, then prove that B is reflexive iff $B^*$ is reflexive. **8.**

4. a) Let T be an operator on a Banach space B. Then prove that T has an inverse $T^{-1}$ if and only if $T^*$ has an inverse $(T^*)^{-1}$ and $(T^*)^{-1} = (T^{-1})^*$. **4**

b) Show that A closed convex subset C of a Hilbert space H contains a unique vector of smallest norm. **8**

c) If M is a linear subspace of a Hilbert space, then show that M is closed $\Leftrightarrow M = M^{\perp\perp}$. **4**

5. a) State and prove Riesz representation theorem for Hilbert space. **8**

b) Show that every orthonormal set in a Hilbert space is contained in some complete orthonormal set. **4**

c) If A is a positive then prove that I+A is non-singular. **4**

6. a) If P and Q are the projections on closed linear subspaces M and N of H. Then prove that PQ is a projection $\Leftrightarrow PQ = QP$. In this case, show that PQ is the projection on $M \cap N$. **8**

b) If T is an operator on H, then prove that the following conditions are all equivalent to one another : **8**

i) $T^*T = I$
ii) $(T_x, T_y) = (x, y)$ for all x and y
iii) $\|T_x\| = \|x\|$ for all x.

7. a) Define : **8**
i) fixed point space.
ii) contraction mapping.
iii) characteristic equation of an operator T
iv) Spectrum

b) State and prove finite dimensional spectral theorem. **8**

8. a) State Picard's theorem. **3**

b) If T is a contraction defined on a complete metric space X, then prove that T has a unique fixed point. **9**

c) If T is a normal operator on a Hilbert space then prove that each eigen space of T reduces T. **4**

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Seat Number

*3032*

# MT - 201
# Advanced Real Analysis
## (Old) (201)

P. Pages : 2

Time : Three Hours Max. Marks : 80

Instructions to Candidates :
1. Do not write anything on question paper except Seat No.
2. Graph or diagram should be drawn with the black ink pen being used for writing paper or black HB pencil.
3. Students should note, no supplement will be provided.
4. Attempt **any five** questions.
5. Figures to the right indicate full marks.

**1.** a) Define complete measure and $\sigma$-finite measure. Show that Lebesgue measure in defined on $\mu$, the class of measurable sets or $\mathbb{R}$, is $\sigma$-finite and couple. **4**

b) If $\mu$ is a $\sigma$-finite measure on a ring $\mathcal{R}$, show that it has a unique extension to the $\sigma$-ring $\mathcal{S}(\mathcal{R})$. **6**

c) In the question 1(b) is the condition of $\sigma$-finite ness of $\mu$ necessary? Justify your answer. **6**

**2.** a) Define convex and strictly convex function let $\psi$ be convex on (a, b) and a<s<t<u<b. If $\psi(a, b)$ denote $\frac{\psi(b)-\psi(a)}{b-a}$ then show that $\psi(s, t) \le \psi(s, u) \le \psi(t, u)$. Also show that if $\psi$ is strictly convex, equality will not occur. **5**

b) State and prove Jensen's in equality. **6**

c) If f and g are in $L'(\mu)$ then show that **5**

i) $\sqrt{|fg|} \in L'(\mu)$

ii) $|f|^p |g|^q \in L'(\mu)$ if $p, q \in (0, 1)$, $p+q=1$.

3. a) If $1 \le p < \infty$ and $\{f_n\}_{n=1}^{\infty}$ is a sequence in $L^P(\mu)$ such that $\| f_n - f_m \|_p \to 0$ as $m, n \to \infty$, prove that there exist a function f and a subsequence $\{n_i\}$ such that $\lim f_{n_i} = f$ a.e. Also $f \in L^P(\mu)$ and $\lim \| f_n - f \|_p = 0$. 7

b) State and prove Egorov's Theorem. 6

c) If $f_n \to f$ in the mean of order $p(p > 0)$, show that $f_n \to f$ in measure. 3

4. a) Let the sequence $\{f_n\}_{n=1}^{\infty}$ be almost uniformly fundamental then show that there exist a measurable function f such that $f_n \to f$ a.u. 6

b) Let $\mu^*$ be the outer measure on $\mathcal{H}(\mathcal{R})$ defined by $\mu$ on $\mathcal{R}$ is contained in $\mathcal{S}^*$, the class of all $\mu^*$-measurable sets in $\mathcal{H}(\mathcal{R})$. 5

c) Show that if $f_n \to f$ in measure then $|f_n| \to |f|$ in measure. 5

5. a) Let $\mu$ be a measure on a measurable space $[\![X, \mathcal{S}]\!]$ and $f : X \to \mathbb{R}_e = \mathbb{R} \cup \{\infty, -\infty\}$ such that $\int f \, d\mu$ is defined. Show that if $\phi(E) = \int_E f \, d\mu$, $E \in \mathcal{S}$, then $\phi$ is a signed measure on $[\![X, \mathcal{S}]\!]$. 6

b) State and prove Jordan decomposition theorem. 6

c) Give an example to show that Hahn decomposition is not unique. 4

6. a) Let $[\![X, \mathcal{S}, \mu]\!]$ be a $\sigma$-finite measure space and $\nu$ a $\sigma$-finite measure on $\mathcal{S}$. Then show that $\nu = \nu_0 + \nu_1$ where $\nu_0, \nu_1$ are measures on $\mathcal{S}$ such that $\nu_0 \perp \mu$ and $\nu_1 \ll \mu$. Is this decomposition unique ? Justify. 6

b) If $\nu_1, \nu_2$ are $\sigma$-finite measures on $[\![X, \mathcal{S}, \mu]\!]$ and $\nu_1 \ll \mu$, $\nu_2 \ll \mu$ then prove that $\frac{d}{d\mu}(\nu_1 + \nu_2) = \frac{d\nu_1}{d\mu} + \frac{d\nu_2}{d\mu} [\mu]$. 5

c) Let $\nu$ be a signed measure on measurable space $[\![X, \mathcal{S}]\!]$. Prove that there exist measures $\nu^+$ and $\nu^-$ on $[\![X, \mathcal{S}]\!]$ such that $\nu = \nu^+ - \nu^-$ and $\nu^+ \perp \nu^-$. 5

7. State and prove Riesz representation theorem. 16

8. State and prove Fubini's theorem for product measures. 6

b) Show that $\sigma$-algebra generated by $\mathcal{A}$ is the smallest monotone class containing $\mathcal{A}$. 6

c) If $\mu$ and $\nu$ are complete measures show that $\mu \times \nu$ need not be complete. 4

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Seat Number

*3033*

# MT-402
# Quantitative Techniques (402)

P. Pages : 3

Time : Three Hours

Max. Marks : 80

Instructions to Candidates :
1. Do not write anything on question paper except Seat No.
2. Graph or diagram should be drawn with the black ink pen being used for writing paper or black HB pencil.
3. Students should note, no supplement will be provided.
4. Attempt **any five** questions.
5. Figures to the right indicate full marks.
6. Scientific calculator is allowed.
7. Statistical table are allowed.

**1.** a) Discuss the rules for construction of network diagrams. Also explain the differences between CPM and PERT. **8**

b) A project consists of following activities. **8**

| Activity | A | B | C | D | E | F | G | H | I | J | K | L |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Predecessor | - | - | A | B | B | C,D | E | E | G | H | J,I | F |
| Duration (Days) | 4 | 1 | 1 | 1 | 6 | 5 | 4 | 8 | 1 | 2 | 5 | 7 |

Draw the network diagram and hence find.

i) Critical path ii) Project duration

ii) Floats for each activity.

**2.** a) A project is given below. **8**

| Job: | : | 1-2 | 2-3 | 3-5 | 7-8 | 5-8 | 6-7 | 4-5 | 1-6 | 2-4 |
|---|---|---|---|---|---|---|---|---|---|---|
| Duration: (Days) | to : | 3 | 6 | 5 | 4 | 1 | 3 | 3 | 2 | 2 |
| | tm : | 6 | 12 | 11 | 19 | 4 | 9 | 6 | 5 | 5 |
| | tp : | 15 | 30 | 17 | 28 | 7 | 27 | 15 | 14 | 8 |

i) Draw network diagram

ii) Find critical path.

iii) Project duration

iv) Find the probability of completing the project before 31 days.

b) Discuss the process of crashing and to get optimal cost and duration of of the project. **8**

3. a) Explain sequencing of n jobs on three machines with assumption involved. Also explain how obtain optimal sequence by Johnson's methods. 8

b) 8

| Book : | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| Composing : | 40 | 90 | 80 | 60 | 50 |
| Printing : | 50 | 60 | 20 | 30 | 40 |
| Binding : Time in days | 80 | 100 | 60 | 70 | 110 |

Find an optimal sequence and hence T min and idle times.

4. a) Explain the following terms: 8

i) EMV
ii) EOL
iii) EVPI
iv) Decision tree.

b) Shrikhand is produced at Rs. 60/kg and sold at Rs. 80/kg. Historical data is given below: 8

| Units sold per day: | 180 | 181 | 182 | 183 | 184 | 185 | 186 |
|---|---|---|---|---|---|---|---|
| No. of days | 2 | 8 | 10 | 40 | 20 | 15 | 5 |

Unused Shrikhand becomes worthless at the end of the day. Find the optinumber of units to be produced and the EVPI.

5. a) i) Explain the Replacement problem and its applications.
ii) Write a short note on group replacement policy. 8

b) The cost of purchasing a machine is Rs. 30,0000 and the running cost is given below. 8

| Year : | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| Cost (Rs.) : | 50,000 | 60,000 | 80,000 | 1,00,000 | 1,30,000 | 1,60,000 | 2,00,000 |

Find the optimal replacement period if the rate of interest is 10% and scrap value is nil.

6. a) What is EOQ? Discuss the steps involved in developing EOQ model. 8

b) For part A, The annual usage was 1000 units, ordering cost = Rs. 400, holding cost= 40% and cost per unit = Rs. 20. The following strategies are available: 8

i) Place 4 orders of equal size each year.
ii) Place the order for 500 units at a time and avail a discount of 10% on cost of items.
iii) EOQ policy.

Which of the above would you recommend? Justify.

7. a) Explain simulation and list reasons for using it. Also discuss the drawbacks of using simulation. **8**

b) A production line turns about 50 trucks/ day. The production data is : **8**

| Production /day: | 45 | 46 | 47 | 48 | 49 | 50 | 51 | 52 | 53 | 54 | 55 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Probability : | 0.03 | 0.05 | 0.07 | 0.1 | 0.15 | 0.2 | 0.15 | 0.1 | 0.07 | 0.05 | 0.03 |

Finished Trucks are transported by trains at the end of the day. If the train capacity is only 51, what is the average number of trucks waiting to be shipped and what will be the average number of empty slots on the train? Use the random numbers:

- 37, 35, 63, 25, 50, 71, 95, 16 for production of next 8 days simulation.

8. Write an explanatory note on the following. **16**

i) Application of Queuing Models.

ii) Single channel Queuing Models

iii) M/M/1 : Quelength finite Model

iv) M/M/1 : Limited Resource Model

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Seat Number

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* 3 0 3 4 *

# MT - 102
# General Topology
## (New) (111102)

**P. Pages : 3**

**Time : Three Hours** **Max. Marks : 80**

Instructions to Candidates :
1. Do not write anything on question paper except Seat No.
2. Graph or diagram should be drawn with the black ink pen being used for writing paper or black HB pencil.
3. Students should note, no supplement will be provided.
4. Attempt **any five** questions.
5. Figures to the right indicate full marks.

**1.** a) Let $\mathcal{J}$ denote the collection consisting of $\mathbb{R}, \phi$ and all infinite open intervals $A_q = (q, \infty)$ with $q \in Q$, the rationals. Is $\mathcal{J}$ a topology on X? Justify. **4**

b) Let $\mathfrak{B}$ *and* $\mathfrak{B}$ be bases for the topologies of and $\mathcal{J}'$ ,respectively, on X. Show that the following are equivalent: **6**
   i) $\mathcal{J}'$ is finer than $\mathcal{J}$.
   ii) For each $x \in X$ and each basis element $B \in \mathfrak{B}$ containing x, there is a basis element $B' \in \mathfrak{B}$ such that $x \in B' \subset B$.

c) Define a sub basis for a topology on a set X. Verify that the topology generated by a sub basis for X is infact, a topology on X. **6**

**2.** a) If $\mathfrak{B}$ is a basis for the topology of X, and $\mathcal{C}$ is a basis for the topology of Y, prove that the collection: $\mathcal{D} = \{B\ x\ C / B \in \mathfrak{B}, C \in \mathcal{C}\}$ is the basis for the topology of X x Y. **6**

b) Consider the subset $Y = [0, 1) \cup \{2\}$ of the real line $\mathbb{R}$. Is {2} an open set in Y (as a subspace) ? Is it open in $\mathbb{R}$ ? **4**

c) Let Y be a subspace of a topological space X. Prove that a set A is closed in Y if and only if it equals the intersection of a closed set in X with Y. **6**

3. a) Let X be a Hausdorff space and let A be a subset of X. Prove that a point x is a limit point of A if and only if every neighbourhood of x contains infinitely many points of A. 6

b) Let A and B denotes the subsets of a topological space X. Determine whether the following equations hold. If an equality fails, justify by an example. 6

i) $\overline{A \cup B} = \overline{A} \cup \overline{B}$

ii) $\overline{A \cap B} = \overline{A} \cap \overline{B}$

c) Prove that for functions $f: \mathbb{R} \to \mathbb{R}$, the $\in - \delta$ definition of continuity implies the open set definition. 4

4. a) Show that the function $F:(-1, 1) \to \mathbb{R}$ defined by $F(x) = \frac{x}{1-x^2}$ is a homeomorphism. 4

b) Define box topology and product topology on the arbitrary product $X = \prod_{\alpha \in J} X_\alpha$ where each $X_\alpha$ is a topological space. State in general that which topology is finer. 6

c) Let $p: X \to Y$ be a quotient map. Let Z be a space and let $g: X \to Z$ be a continuous map that is constant on each set $p^{-1}(\{y\})$ for $y \in Y$. Then g induces a continuous map $f: Y \to Z$ such that fop=g. Prove it. 6

5. a) Let d and d' be two metrics on the set X ; let $\mathcal{T}$ and $\mathcal{T}'$ be the topologies they induce, respectively. Then show that $\mathcal{T}'$ is finer than $\mathcal{T}$ if and only if for each x in X and each $\in > 0$, there exists a $\delta > 0$ such that $B_{d'}(x, \delta) \subset B_d(x, \in)$. 6

b) Let X be a topological space and let $A \subset X$. IF there is a sequence of points of A converging to x, then show that $x \in \overline{A}$. Show also that the converse hold if X is metrizable. 6

c) Is the space $\mathbb{R}_l$ connected ? Justify your answer. 4

6. a) State and prove intermediate value theorem. 6

b) Prove that the space X is locally connected if and only if for every open set U of X, each component of U is open in X. 6

c) Find the components and path components of the subspace $Y = [-1, 0) \cup (0, 1]$ of the real line $\mathbb{R}$. 2

d) True or false : The real line $\mathbb{R}$ is compact. **2**

7. a) Let Y be a subspace of X. Prove that Y is compact if and only if every open covering of Y by open sets in X contains a finite sub collection covering Y. **6**

b) Prove that compactness implies limit point compactness but not conversely. **6**

c) Define completely regular space. Give an example. **4**

8. a) Prove that every compact Hausdorff space is normal. **4**

b) State and prove Urysohn lemma. **12**

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Seat Number

*3035*

# MT-102
# Topology (Old)
# (102)

P. Pages : 2

Time : Three Hours | Max. Marks : 80

Instructions to Candidates :
1. Do not write anything on question paper except Seat No.
2. Graph or diagram should be drawn with the black ink pen being used for writing paper or black HB pencil.
3. Students should note, no supplement will be provided.
4. Attempt **any five** questions.
5. Figures to the right indicate full marks.

1. a) Prove that the lower limit topology is strictly finer than the standard topology on IR — 4

   b) If $\mathfrak{B}$ be a basis for the topology $\mathcal{T}$ on X, prove that $\mathcal{T}$ equals the collection of all unions of elements of $\mathfrak{B}$. — 6

   c) If A, B and $A_\alpha$ denote subsets of X, then prove the following : — 6

   i) $\overline{A \cup B} = \overline{A} \cup \overline{B}$ and

   ii) $\overline{\bigcup_\alpha A_\alpha} \supset \bigcup_\alpha \overline{A}_\alpha$

2. a) Show that the map $F:(-1,1)\to IR$ defined by $F(x)=\dfrac{x}{1-x^2}$ is a homeomorphism. — 4

   b) Let X be a topological space. Suppose that $\mathcal{C}$ is a collection of open sets of X such that for each $x \in X$ and each open set U of X, there is $c \in \mathcal{C}$ such that $x \in C \subset U$. Show that $\mathcal{C}$ forms a basis for the topology of X. — 6

   c) State and prove gluing lemma. — 6

3. a) Prove that a continuous image of connected space is connected. — 6

   b) Prove that the rational Q are not connected. — 4

c) If A is a subspace of X and B is a subspace of Y, prove that the product topology on $A \times B$ is the same the topology $A \times B$ inherits as a subspace of $X \times Y$. 6

4. a) With usual notations prove that $\overline{A} = A \cup A'$. 6

b) Prove that every simply ordered set is a Hausdorff space in the order topology. 4

c) Let $p: X \to Y$ be a quotient map. Let Z be a space and let $g: X \to Z$ be a continuous map that is constant on each set $p^{-1}:(\{y\})$ for $y \in Y$, Prove that g induces a continuous map $f: Y \to Z$ such that $fop = g$. 6

5. a) Prove that a closed subspace of a normal space is normal. 6

b) Prove that a completely regular space is regular. 6

c) Construct an example of a space that satisfies $T_1$ – axiom but it is not Hausdorff. 4

6. a) Prove that every metrizable space is normal. 6

b) Prove that every second countable space is first countable. Is the converse true? Justify. 6

c) Is the real line IR with standard topology compact? Is it locally compact? Justify. 4

7. a) Let X be a Hausdorff space. Prove that X is locally compact at $x \in X$ if and only if for every neighbourhood U of x, there is a neighbourhood V of x such that $\overline{V}$ is compact and $\overline{V} \subset U$. 6

b) Show that the one point compactification of IR is homeomorphic to the circle $S^1$. 6

c) Let $X = \{a,b,c\}$ and $\mathcal{T}_1 = \{\phi, X, \{a\}\, \{b,c\}\}$ and $\mathcal{T}_2 = \{\phi, X, \{a\}\, \{a,b\}\}$ find the smallest topology containing $\mathcal{T}_1$ and $\mathcal{T}_2$. Also find the largest topology contained in $\mathcal{T}_1$ and $\mathcal{T}_2$. 4

8. a) State and prove Tychonoff theorem. 12

b) For functions $f: IR \to IR$, prove that the $\epsilon - \delta$ definition of continuity implies the open set definition. 4

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Seat Number

* 3 0 3 6 *

# MT-302
# Fluid Mechanics
## (302)

**P. Pages : 2**

**Time : Three Hours** **Max. Marks : 80**

Instructions to Candidates :
1. Do not write anything on question paper except Seat No.
2. Graph or diagram should be drawn with the black ink pen being used for writing paper or black HB pencil.
3. Students should note, no supplement will be provided.
4. Attempt **any five** questions.
5. Figures to the right indicate full marks.

**1.** a) Explain different types of flows. **8**

b) The velocity components for a two dimensional fluid system can be given in the Eulerian system by $u = 2x + 2y + 3t,\ v = x + y + \frac{1}{2}t$. Find the displacement of a fluid particle in the Lagrangian system. **8**

**2.** a) Air, obeying Boyle's law is in motion in a uniform tube of small section, prove that it ρ be the density and ν the velocity at a distance x from a fixed point at time $t_i$ $\frac{\partial^2 \rho}{\partial t^2} = \frac{\partial^2}{\partial x^2}\left[\rho\left(v^2 + k\right)\right]$ **6**

b) Write a note on **6**
i) Boundary conditions (Kinematical)
ii) Boundary conditions (Physical) and
iii) Boundary surface.

c) Show that $\left(x^2/a^2\right)f(t) + \left(y^2/b^2\right)\phi(t) + \left(z^2/c^2\right)4(t) = 1$ is a possible form of the boundary surface if $f(t)\cdot\phi(t)\cdot 4(t) = 1$. **4**

**3.** a) With usual notations, derive the equation of motion : **8**
$$\frac{\partial \bar{q}}{\partial t} - \bar{q} \times \text{curl}\ \bar{q} = \bar{F} - \frac{1}{\rho}\nabla p - \frac{1}{2}\nabla q^2.$$

b) An infinite fluid in which a spherical hollow of radius a is initially at rest under the action of no forces. If a constant pressure P is applied at infinity, show that the time of filling up the cavity is **8**
$$a\left(\frac{\pi\rho}{6p}\right)^{1/2}\frac{\Gamma(5/6)}{\Gamma(4/3)}.$$

4. a) State and prove Green's theorem and hence show that the cyclic irrotational motion is impossible in a liquid bounded by rigid boundaries. **8**

b) The velocity field is given by $\overline{q}=\frac{(-\bar{i}y+\bar{j}x)}{x^2+y^2}$. Determine whether the flow is irrotational. Calculate the circulation round a **8**

i) Square with corners at (1,0), (2,0), (2,1), (1,1);

ii) Unit circle with centre at the origin.

5. a) State and prove Blasius theorem. **8**

b) Show that $\phi=(x-t)(y-t)$ represents the velocity potential of an incompressible two dimensional fluid. Show that the stream lines are given by $(x-t)^2-(y-t)^2$ = constant and the path lines are given by $\log(x-y)=\frac{1}{2}[(x+y)-a(x-y)^{-1}]+b$. **8**

6. a) State and prove Kelvin's circulation theorem. **8**

b) In irrotational motion two dimensions, Prove that **8**

$$\left(\frac{\partial q}{\partial x}\right)^2+\left(\frac{\partial q}{\partial y}\right)^2=q\nabla^2 q.$$

7. a) In the region bounded by a fixed quadrantal arc and its radii, deduce the motion due to a source and an equal sink situated at the ends of one of the bounding radii. Show that the stream line leaving either end at an angle $\alpha$ with the radius is $\gamma^2\sin(\alpha+\theta)=a^2\sin(\alpha-\theta)$ **6**

b) Describe plane Poiseuille flow and hence derive the expressions for the velocity and wall shear stress. **10**

8. a) Explain steady laminar flow between two parallel plates and hence derive the expressions for velocity and skin friction. **8**

b) Water at 70°C flows between two large parallel plates at a distance 1/16 inch apart. If the average velocity is $\frac{1}{2}$ft/sec; evaluate. **8**

i) The maximum velocity.

ii) The pressure drop.

iii) The wall shearing stress

iv) The frictional coefficient.

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Seat Number

* 3 0 3 7 *

# MT-202
# Complex Variables
## (New) (111202)

P. Pages : 2

Time : Three Hours

Max. Marks : 80

Instructions to Candidates :
1. Do not write anything on question paper except Seat No.
2. Graph or diagram should be drawn with the black ink pen being used for writing paper or black HB pencil.
3. Students should note, no supplement will be provided.
4. Attempt **any five** questions.
5. Figures to the right indicate full marks.

1\. a) If $\Sigma a_n (z-a)^n$ is a given power series with radius of convergence R, then prove that $R = \lim \left|\frac{a_n}{a_{n+1}}\right|$ if this limit exists. **8**

b) If G is open and connected and $f: G \to \mathbb{C}$ is differentiable with $f'(z) = 0$ for all z in G, then prove that f is constant. **5**

c) Show that $e^z$ is periodic with periods multiples of $2\pi i$. **3**

2\. a) If $f: G \to \mathbb{C}$ is analytic then prove that f preserves angles at each point $Z_0$ of G where $f'(Z_0) \neq 0$. **8**

b) Define function of bounded variation. Let $\gamma: [a.b] \to \mathbb{C}$ be of bounded variation. Prove that : **8**

i) If P and Q are partitions of [a, b] and $P \subset Q$ then $v(\gamma;P) \leq v(\gamma;Q)$.

ii) If $\sigma: [a,b] \to \mathbb{C}$ is also of bounded variation and $\alpha, \beta \in \mathbb{C}$, then $\alpha\gamma + \beta\sigma$ is also of bounded variation and -----------
$V(\alpha\gamma + \beta\sigma) \leq |\alpha| V(\gamma) + |\beta| v(\sigma)$.

3\. a) If $\gamma$ is piecewise smooth and $f: [a,b] \to \mathbb{C}$ is continuous then prove that $\int_a^b f d\gamma = \int_a^b f(t)\gamma'(t)\, dt$. **8**

b) Evaluate : **8**

i) $\int_\gamma \frac{sinz}{z^3}\,dz$ where $\gamma(t) = e^{it}, 0 \le + \le 2\pi$

ii) $\int_\gamma \frac{e^z - e^{-z}}{z^n}\,dz$ where x is positive integer and $\gamma(t) = e^{it}, 0 \le t \le 2\pi$.

**4.** a) State and prove maximum modulus theorem. **6**

b) If $\gamma: [0,1] \to \not{c}$ is a closed rectifiable curve and $a \notin \{\gamma\}$ then prove that $\frac{1}{2\pi i}, \int_\gamma \frac{dz}{z-a}$ is an integer. **6**

c) Let $\gamma$ be a closed rectifiable curve in $\not{c}$ anda $\notin \{\gamma\}$. Show that for $n \ge 2, \int_\gamma (z-a)^{-n}\,dz = 0$. **4**

**5.** a) State and prove open mapping theorem. **8**

b) Let z = a be an isolated singularity of f and **8**

$$f(z) = \sum_{-\infty}^{\infty} a_n (z-a)^n$$

be its Lurent expansion in ann (a; O, R). Prove that

i) z = a is removable singularity iff $a_n = 0$ for n < -1

ii) z = a is a pole of order in iff $a_{-m} \neq 0$ *and* $a_n = 0$ for $n \le -(m+1)$.

**6.** a) State and prove Residue theorem. **8**

b) Show that **8**

$$\int_{-\infty}^{\infty} \frac{x^2}{1+x^4}\,dx = \frac{\pi}{\sqrt{2}}.$$

**7.** a) State and prove Schwarz's Lemma. **8**

b) Define convex function. Prove that a function $f:[a,b] \to \mathbb{R}$ is convex iff the set $A = \{(x,y)/a \le x \le b \text{ and } f(x) \le y\}$ is convex. **8**

**8.** a) State and prove Arzela – Ascoli theorem. **8**

b) State and prove the Riemann mapping theorem. **8**

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Seat Number

* 3 0 3 8 *

# MT-202
# Complex Analysis
## (Old) (202)

**P. Pages : 2**

**Time : Three Hours** **Max. Marks : 80**

Instructions to Candidates :
1. Do not write anything on question paper except Seat No.
2. Graph or diagram should be drawn with the black ink pen being used for writing paper or black HB pencil.
3. Students should note, no supplement will be provided.
4. Attempt **any five** questions.
5. Figures to the right indicate full marks.

**1.** a) Let f and g be analytic functions on G and $\Omega$ respectively and suppose that $f(G) \subset \Omega$. Prove that gof is analytic on G and that $(gof)'(z) = g'(f(z)).f'(z)$. **8**

b) Let u and v be real valued functions defined on a region G and suppose that u and v have continuous partial derivatives. Prove that $f: G \longrightarrow \mathbb{C}$ defined by $f(z) = u(z) + iv(z)$ is analytic if and only if u and v satisfy the Cauchy - Riemann equations. **8**

**2.** a) Let $z_1, z_2, z_3, z_4$ be four distinct points in $\mathbb{C}_\infty$. Prove that $(z_1, z_2, z_3, z_4)$ is a real number if and only if all the four points lie on a circle. **8**

b) Find the following cross ratios: **4**
   i) $(7 + i, 1, 0, \infty)$
   ii) (0, 1, i, -1)

c) Define conformal map. If $f(z) = e^z$, then f is conformal throughout $\mathbb{C}$. Check it. **4**

**3.** a) If S is a Mobius transformation, prove that it is the combination of translations, dilations and the inversion. **6**

b) If $\gamma$ is piecewise smooth and $f: [a, b] \rightarrow \mathbb{C}$ is continuous, prove that $\int_a^b f d\gamma = \int_a^b f(t)p'(t)$ dt. **7**

c) Define $\gamma: [0, 2\pi] \to ¢$ by $\gamma(t)$= exp (int), where $\gamma$ is some integer (Positive, negative or zero). Show that $\int_\gamma \frac{1}{z}\, d_z = 2\pi in.$ 3

4. a) With usual notations prove that 8

$$\int_0^{2\pi} \frac{e^{is}}{e^{is} - z}\, ds = 2\pi, |z| < 1.$$

b) State and prove Liouville's theorem. 8

5. a) State and prove Cauchy's estimate. 6

b) State and prove open mapping theorem. 6

c) Give Laurent series expansion of $f(z) = \frac{1}{z(z-1)(z-2)}$ in the annulus (0; 1, 2) 4

6. a) State and prove residue theorem. 6

b) Show that $\int_{-\infty}^{\infty} \frac{x^2}{1+x^4}\, dx = \frac{\pi}{\sqrt{2}}$. 8

c) State the fundamental theorem of algebra. 2

7. a) Prove that C (G, Ω) is a complete metric space. 8

b) State and prove Schwarz's lemma. 8

8. a) State and prove Rouche's theorem. 8

b) State and prove Montel's theorem. 8

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Seat Number

* 3 0 3 9 *

# MT-403
# Commutative Algebra
# (403)

**P. Pages : 3**

**Time : Three Hours** **Max. Marks : 80**

Instructions to Candidates :
1. Do not write anything on question paper except Seat No.
2. Graph or diagram should be drawn with the black ink pen being used for writing paper or black HB pencil.
3. Students should note, no supplement will be provided.
4. Attempt **any five** questions.
5. Figures to the right indicate full marks.

**1.** a) If M is an R - module and $0 \to N' \xrightarrow{f} N \xrightarrow{g} N'' \to O$ is an exact sequence, prove that the induced sequence. **8**

$0 \to H_{om_R}(M,N') \xrightarrow{f^*} H_{om_R}(M,N) \xrightarrow{g^*} H_{om_R}(M,N'')$ is exact. Is $g^*$ on to? Why?

b) For any R - modules M and N, show that **6**

$0 \to M \xrightarrow{i} M \oplus N \xrightarrow{\Pi} N \to 0$ is split exact sequence, where $i(x) = x$ and $\pi(x + y) = y$, for all $x \in M$, $y \in N$.

c) Define projective R - module. Give an example. **2**

**2.** a) Define tensor product of R - modules and prove that the tensor product of two R - modules exists and is unique. **10**

b) For R - modules M, N, K prove that $(M \oplus N) \otimes K \cong (M \otimes K) \oplus (N \otimes K)$. **6**

**3.** a) Prove that an R -module M is faith fully flat if and only if M is flat and for each maximal ideal I of R, $IM \neq M$. **6**

b) Let $0 \rightarrow M' \xrightarrow{f} M \xrightarrow{g} M'' \rightarrow 0$ be an exact sequence of R-modules. For any R- modules N, prove that the tensored sequence. **6**

$M' \otimes N \xrightarrow{f^*} M \otimes N \xrightarrow{g^*} M'' \otimes N \rightarrow 0$ is exact.

c) Show that $\mathbb{Z}_2$ is not a flat $\mathbb{Z}$ - module. **4**

**4.** a) Let $0 \rightarrow M' \rightarrow M \rightarrow M'' \rightarrow 0$ be an exact sequence of R-modules. Prove that M is Artinian if and only if M' and M" are Artinian. **6**

b) Let M, N be R - modules and $f: M \rightarrow N$ an R - homomorphism. Prove that the following conditions are equivalent. **6**

i) f is injective.

ii) $f_P : M_P \rightarrow N_P$ is injective, for all prime ideals P of R.

iii) $f_J : M_J \rightarrow N_J$ is injective, for all maximal ideals J of R.

c) Is $\mathbb{Z}$ an Artinian ring? Justify. **4**

**5.** a) State and prove Going Up Theorem. **6**

b) If R is an Artinian ring, show that the Jacobson radical of R is nilpotent. **6**

c) Let $R \subseteq S$ be a ring extension. Show that the set of all elements of S integral over R is a subrings of S containing R. **4**

**6.** a) Show that every UFD is integrally closed. **6**

b) Let S be an integral extension of R, J be any ideal of S and $I = J \cap R$. Prove that $\frac{S}{J}$ is integral over $\frac{R}{I}$. **4**

c) Let R be a Dedekind domain with only finitely many prime ideals. Prove that R is a PID. **6**

**7.** a) Let R be a Noetherian local domain with unique maximal ideal $I \neq 0$ and K the quofient field of R. **10**
Prove that the following are equivalent:

i) R is a PID.

ii) I is a principal ideal.

iii) R is integrally closed and every nonzero prime ideal of R is maximal.

b) Define local ring and valuation ring. Prove that a valuation ring V of K is a local ring. **6**

**8.** a) Let R be a local domain. Prove that every non zero fractionary ideal of R is invertible if and only if R is a DVR. **8**

b) Show that the integral closure of $\mathbb{Z}$ in $\mathbb{Q}[i]$ is $\mathbb{Z}[i]$. **4**

c) Define discrete valuation ring. Give an example. **4**

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Seat Number

MT-103
# Algebra
## (New) (111103)

P. Pages : 2

Time : Three Hours

Max. Marks : 80

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Instructions to Candidates :
1. Do not write anything on question paper except Seat No.
2. Graph or diagram should be drawn with the black ink pen being used for writing paper or black HB pencil.
3. Students should note, no supplement will be provided.
4. Attempt **any five** questions.
5. Figures to the right indicate full marks.

**1.** a) Let G be a group and a $\in$G. Prove that the number of elements in the conjugate class C(a) is equal to the index of the normalizer N (a) of a in G. **6**

b) Find the conjugate classes of $S_4$ and verify class equation. **6**

c) If G is a finite group with just two conjugate classes then show that $0(\in) = 2$. **4**

**2.** a) State and prove first sylow theorem. **6**

b) Show that any group of order 56 can not be simple. **6**

c) Let G be a finite group and P be a sylow p-subgroup. If H is any subgroup of G of order $p^i$ $(i \geq 1)$ with $H \subset N(P) = \{a \in G \mid aP = Pa\}$ then show that $H \subset P$. **4**

**3.** a) Define and give one example of each of the following. **6**

i) A solvable group

ii) A normal series

iii) A composition series.

b) Show that Sn is not solvable for $n \geq 5$. **8**

c) True or false? Justify. **2**

If every proper subgroup of G is solvable then G is solvable.

4. a) Prove that every Euclidean domain is a PID. 8

b) Show that every prime element is irreducible. 4

c) In the ring $R = \{a + b\sqrt{-5} \mid a, b \in \mathbb{Z}\}$, investigate whether the following elements are irreducible? If not, write down all their proper divisors. 4

i) $3 + \sqrt{-5}$ ii) $4 + 3\sqrt{-5}$

5. a) Let R be a PID prove that every a∈R, which is not a unit can be expressed as a product of irreducible elements. 6

b) Show that in a PID, every ideal is contained in a maximal ideal. 4

c) Show that every PID is a Noetherian ring. 6

6. a) Show that every Noetherian ring R which is an integral domain R is a factorisation domain. 6

b) If F is a field then prove that F [x] is a Euclidean domain. 6

c) Show that $\sqrt[4]{7}$ is irrational. 4

7. a) Let R be a commutative ring with 1. Prove that following conditions are equivalent. 10

i) Any ascending chain of ideals of R is stationary.

ii) Every ideal of R is finitely generated.

iii) Every non-empty collection of ideals of R has a maximal element.

b) True or false? Justify. 4

i) Quotient ring of a PID is a PID.

ii) Product of two PID'S is a PID.

c) What are the units in $\mathbb{Z}_7$ [x] ? 2

8. a) State and prove Hilbert basis theorem. 12

b) Show that a homomorphic image of a Noetherian ring is Noetherian. 4

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Seat Number

*30041*

# MT-103
# Abstract Algebra
## (Old) (103)

**P. Pages : 2**

**Time : Three Hours** **Max. Marks : 80**

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Instructions to Candidates :

1. Do not write anything on question paper except Seat No.
2. Graph or diagram should be drawn with the black ink pen being used for writing paper or black HB pencil.
3. Students should note, no supplement will be provided.
4. Attempt **any five** questions.
5. Figures to the right indicate full marks.

**1.** a) Prove that the group $G = H \times K$ is abelian if and only if both H and K are abelian. **4**

b) Prove that the number of distinct conjugate classes of $S_n$ is p(n). **6**

c) Find the conjugate classes of $Q_8$ and verify the class equation for it, where **6**

$$Q_8 = \{\pm 1, \pm i, \pm j, \pm k \mid i^2 = j^2 = k^2 = -1, i \cdot j = k, j \cdot k = i, k \cdot i = j\}$$

**2.** a) State and prove Sylow first theorem. **8**

b) Show that the group of order 108 cannot be simple. **6**

c) True of false : Every group of order 77 is cyclic. Justify. **2**

**3.** a) Prove that a group of prime power order is solvable. **6**

b) Define composition series. Show that any non-trivial finite group has a composition series. **6**

c) Prove or disprove : If every proper subgroup of G is solvable then G is solvable. **4**

4\. a) Let R be an integral domain with unit element. Prove that and element $p \in R$ is prime if and only if the ideal $R_P = \{xp \mid x \in R\}$ is prime. 6

b) Show that $Z[\sqrt{-5}]$ is not a Euclidean domain. 4

c) Let R be an Euclidean domain. Prove that any $a \in R - \{0\}$ which is not unit can be expressed as a product of irreducible elements. 6

5\. a) Prove that in a PID every nonzero prime ideal is maximal. 4

b) Give an example to show that 6
   i) Every F.D. need not be U.F.D.
   ii) Every U.F.D. need not be P.I.D.
   iii) Every P.I.D. need not be E.D.

c) If R is a u.f.d., prove that any two nonzero elements of R have a g.c.d. 6

6\. a) If F is a field, prove that F[x] is an Euclidean domain. 6

b) Let F be a field and let $f(x) \in F[x]$ be a polynomial of degree n. Prove that f(x) has atmost n distinct roots in F. 6

c) Find the g.c.d. of $f(x) = x^3 - 2x^2 + 6x - 5$ and $g(x) = x^2 - 2x + 1$ in Q[x], where Q is the field of rationals. 4

7\. a) State and prove Gauss lemma. 8

b) Prove that the quotient ring of Noetherian ring is Noetherian. 4

c) Prove or disprove : A product of two P.I.D.'s is a P.I.D. 4

8\. a) State and prove Hilbert basis theorem. 8

b) Show that $\frac{F[x]}{\langle 2, x \rangle}$ is Noetherian, where F is a field. 6

c) Give an example of a Noetherian ring which is not a P.I.D. 2

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Seat Number

* 3 0 4 2 *

# MT-303
# Advanced Algebra
## (303)

**P. Pages : 3**

**Time :Three Hours** **Max. Marks : 80**

Instructions to Candidates :
1. Do not write anything on question paper except Seat No.
2. Graph or diagram should be drawn with the black ink pen being used for writing paper or black HB pencil.
3. Students should note, no supplement will be provided.
4. Attempt **any five** questions.
5. Figures to the right indicate full marks.
6. All rings considered are commutative with identity element.

**1.** a) Let N, K be submodules of an R - module M. Show that $\text{ann}\left(\frac{N+K}{N}\right)=\{a \in R \;:\; ax \in N,\ \forall x \in K\}$. **5**

b) Let N, K be submodules of an R - module M and $K \subseteq N$. Prove that $\frac{M}{N} \cong \frac{M/K}{N/K}$. **5**

c) Let M be a simple R - module and $f: M \to N$ be an R - module homomorphism. Prove that **6**

i) f (M) is a simple submodule of N.

ii) if $f(M) \neq \{0\}$, then f is one-one.

**2.** a) Let M be a free R - module of rank 1, where R is PID. If N is a submodule of M, then prove that N is free of rank 0 or 1. **6**

b) Let $N_1$, $N_2$ be submodules of an R - module M. If $M = N_1 \oplus N_2$ then show that $\frac{M}{N_1} \cong N_2$. **4**

c) True or false? Justify. 6

i) $\mathbb{Z}_6$ is faithful $\mathbb{Z}_6$ - module

ii) $\mathbb{Z}_6$ is torsion $\mathbb{Z}_6$ - module

iii) $\mathbb{Z}_6$ is torsion $\mathbb{Z}$ - module.

**3.** a) Prove that every finitely generated torsion free module over PID is free. 8

b) Define period of an element in a module over a PID R. if M be a module over a PID R & $x \in M$, then prove that period of x is a if and only if ann (x) = <a>. 5

c) Prove or disprove: Let M be a module over a ring R. Then $tor_R$ (M) is a submodule of M. 3

**4.** a) Let M be a finitely generated torsion module over a PID R with exponent $P^r$ $(r \geq 1, P \text{ is prime element})$. Prove that 10

$$M \cong \frac{R}{\langle P^{r_1} \rangle} \oplus \frac{R}{\langle P^{r_2} \rangle} \oplus \ldots\ldots\ldots\ldots \oplus \frac{R}{\langle P^{r_k} \rangle}, \text{ where } r_1 \geq r_2 \geq \ldots\ldots\ldots\ldots \geq r_k \geq 1.$$

b) Prove that any two finitely generated modules over a PID R are isomorphic if and only if they have same rank and same invariant factors. 6

**5.** a) Let V be finite dimensional F - vector space and $T: V \to V$ be a linear map. For $v \cong V$, $f(x) \in f\ [x]$, define $f(x)\,v = f(T)v$. Prove that V is finitely generated torsion f[x] - module with exponent equal to minimum polynomial of T. 6

b) Find the Jordan from of the 6 x 6 matrix $A = \begin{bmatrix} -1 & -2 & 6 \\ -1 & 0 & 3 \\ -1 & -1 & 4 \end{bmatrix}$. 5

c) Find all possible Jordan forms for 6x6 matrices with $(x - 2)^3\,(x+3)$ as a minimum polynomial. 5

**6.** a) Let $T: V \to V$ be a linear map with minimum polynomial $q(x) = a_0 + a_1 x + \ldots\ldots\ldots a_{n-1} x^{n-1} + x^n$. If V is cyclic f [x] - module then prove that three exists a basis of V such that the matrix of T is C (q(x). 8

b) Find all possible rational canonical forms for 6 x 6 matrices with $(x-2)(x+2)^3$ as minimum polynomial. **8**

**7.** a) Let R be a ring. Prove that N (R) = ∩ {P/P is a prime ideal of R}. **6**

b) Let $I_1, I_2, \ldots\ldots I_n$ be painwise comaximal ideals of a ring R. **5**

Show that $\bigcap_{i=1}^{n} I_i = \coprod_{i=1}^{n} I_i$.

c) If R is a local ring, then show that every element of 1 + M is unit in R, Where M is a maximal ideal of R. **5**

**8.** a) Prove that a proper submodule N of an R - module M is primary if and only if $a \in R, x \in M, ax \in N \Rightarrow$ either $x \in N$ or $a^n M \subseteq N$, for some $n \geq 1$. **6**

b) Let N be primary submodule of an R - module M. Show that $r_m$ (N) is a prime ideal of R. **4**

c) Prove that every proper submodule of a Noetherian R - module M has a reduced primary decomposition. **6**

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Seat Number

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* 3 0 4 3 *

# MT-203
# Linear Algebra
## (New) (111203)

P. Pages : 3

Time : Three Hours

Max. Marks : 80

Instructions to Candidates :
1. Do not write anything on question paper except Seat No.
2. Graph or diagram should be drawn with the black ink pen being used for writing paper or black HB pencil.
3. Students should note, no supplement will be provided.
4. Attempt **any five** questions.
5. Figures to the right indicate full marks.
6. All rings considered are commutative with identity element.

1. a) If N, K are submodules of an R - module M, then show that N + K and N∩K are also submodules of M. **6**

   b) Let $f: M \rightarrow N$ be an R - module homomorphism. Prove that $\frac{M}{\ker(f)} \cong I_m(f)$. **6**

   c) Let M be a simple R - Module and $f: m \rightarrow M$ be an R - module homomorphism. If $f \neq 0$, then show that f is an isomorphism. **4**

2. a) Let $m_1 M_2, \ldots, M_n$ be submodules of an R - module M. Prove that $M = M_1 \oplus M_2 \oplus \ldots \oplus M_n$ if and only if each $x \in M$ can be uniquely written an $x = x_1 + x_2 + \ldots + x_n$, where $x_i \in M_i$. **6**

   b) Show that an R - module M is finitely generated if and only if $M \cong \frac{R^n}{I}$, for some ideal I of $R^n$ and natural number n. **6**

   c) Prove or Disprove: Submodule of a cyclic module over a ring R is cyclic. **4**

3. a) Let M be a module over a PID R. Prove that $\frac{M}{tor_R(M)}$ is a torsion free R - module. 5

b) Define exponent of a module. Let M be a module over a PID R. Prove that $exp_R(M) = a$ if and only if $ann_R(M) = \langle a \rangle$. 7

c) Prove or disprove: for any R - module M, $tor_R(M)$ is a submodule of M. 4

4. a) Let M be a module over a PID R. With exponent ab, where (a, b) = 1. Prove that $M = M_a \oplus M_b$. where $M_a = \{x \in M : ax = 0\}$ and $M_b = \{x \in M : bx = 0\}$. 8

b) If two finitely generated modules over a PID R are isomorphic then prove that they have same rank and same invariant factors. 5

c) Find the number of abelian groups of order 160000. 3

5. a) Let V be a finite dimensional F - vector space and $T : V \to V$ be a linear map. $U \in V$, $f(x) \in F[x]$ define $f(x)v = f(t)v$. Prove that V is a finitely generated F [x] - module with exponent equal to the minimum polynomial of T. 6

b) Let V be a finite dimensional F - vector space and $T : V \to V$ be a linear map. Let $V = V_1 \oplus V_2$, where $V_1, V_2$ are invariant subspaces under T and $T_i = T/V_i$, i = 1,2. If $P_i(x)$ is the minimum polynomial of $T_i$, i = 1,2, then show the minimum polynomial of T = $\ell$..c.m.$\{P_1(x), P_2(x)\}$. 5

c) Let A $[a_{ij}]_{6x6}$ be a matrix over $\mathbb{C}$. If $(x-3)^2 (x-5)^3$ is the minimum polynomial A, then find all possible Jordan canonical forms of A. 5

6. a) State and prove the theorem on Rational Canonical form. 10

b) Find all possible rational canonical forms of 6x6 matrices with $(x-3)^2 x^2$ as a minimum polynomial. 6

7. a) Prove that every non-unit of a ring R is contained in some maximal ideal of R. 4

b) Let I, J be ideals of a ring R. Prove that I, J are co-maximal ideals of R if and only if $\sqrt{I}$, $\sqrt{J}$ are co-maximal ideals. 6

c) Prove that the set of all non-units of a ring R forms an ideal of R if and only if R has a unique maximal ideal. 6

8. a) Let M be a finitely generated R - Module and I an ideal of R with $IM = M$. Prove that there exists $a \in I$ such that $(1 + a) M = 0$. 7

b) Prove that every proper submodule of a Noetherian R - module M has a reduced primary decomposition. 5

c) Let N be a submodule of an R - module M. Show that 4

$r_m(N) = \left\{ a \in R : \text{The homothety } \lambda_a : \frac{M}{N} \to \frac{M}{N} \text{ is not injective} \right\}$.

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Seat Number

* 3 0 4 4 *

# MT-203
# Field Theory
## (Old) (203)

P. Pages : 2

Time : Three Hours

Max. Marks : 80

Instructions to Candidates :
1. Do not write anything on question paper except Seat No.
2. Graph or diagram should be drawn with the black ink pen being used for writing paper or black HB pencil.
3. Students should note, no supplement will be provided.
4. Attempt **any five** questions.
5. Figures to the right indicate full marks.

1. a) Define simple extension. Give an example of simple extension. 3

   b) If L/K and K/F are finite extensions, show that L/F is a finite extension. 6

   c) Prove that every finite extension is algebraic. Is the converse true? Justify by an example. 7

2. a) Find the degree of the field extension Q $(\sqrt{2}, \sqrt{3}, \sqrt{5})$ | Q. 4

   b) If ch.F=0 and f (x) $\epsilon$F [x] is a monic irreducible polynomials show that f(x) is separable. 6

   c) Show that every algebraically closed field is perfect. 6

3. a) Show that the splitting field of $\overset{p}{x} - 1$ in Q [x], where P is prime, is of degree p-1 over Q. 5

   b) If K/F is a finite separable extension, prove that K = F $(\alpha)$, for some $\alpha \in$ K. 7

   c) Prove or disprove : Q $(\sqrt[3]{2})$ is normal extension. 4

4. a) Prove that **any two** finite fields with same number of elements are isomorphic. 5

b) Find a field with four elements. 4

c) Let F be a finite field. Show that $F^* = F - \{0\}$ is a cyclic group for multiplication. 7

5. a) State and prove Artin's theorem. 8

b) Find the order of the Galois group of the splitting field of $x^3-10$ over Q ($\sqrt{-3}$). 4

c) If K/F is Galois extension and if L/F is any finite extension, prove that [KL : L] divides [K :F]. 4

6. a) Let K/F be a finite extension where F is a finite field. Prove that K/F is a Galois extension with G (K/F) cyclic. 7

b) What is the splitting field of $x^2+x+1 \in \mathbb{Z}_2[x]$? Is it radical extension of $\mathbb{Z}_2$ ? why? 4

c) Find the auto morphisms of Q ($\sqrt{2}, \sqrt{3}$). 5

7. a) Prove that it is impossible to trisect the angle 60° by ruler and compass. 5

b) If ch $F \neq 2, 3$ and $f(x) \in F[x]$ is such that deg. $f(x) \leq 4$, show that f (x) is solvable by radicals. 5

c) Prove that any two purely transcendental extensions over F of the same degree are F-isomorphic. 6

8. a) Let K/F be a finitely generated transcendental extension. Prove that it has a transcendental base. 8

b) Let F be a perfect field and let K/F be a finitely generated extension. Prove that K/F is separably generated. 8

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Seat Number

*3045*

# MT-404
# Graph Theory
# (441)

**P. Pages : 2**

**Time :Three Hours** **Max. Marks : 80**

Instructions to Candidates :
1. Do not write anything on question paper except Seat No.
2. Graph or diagram should be drawn with the black ink pen being used for writing paper or black HB pencil.
3. Students should note, no supplement will be provided.
4. Attempt **any five** questions.
5. Figures to the right indicate full marks.

**1.** a) Prove that every set of six people contains (at least) three mutual acquaintances or three mutual strangers. **6**

b) Prove or disprove :
The complement of a simple disconnected graph must be connected. **4**

c) Define girth of a graph and Petersen graph. Show that the Petersen graph has girth five. **6**

**2.** a) Show that every u, v-walk contains a u,v-path. **4**

b) Show that for a connected non-trivial graph with exactly $2_k$ odd vertices, the minimum number of trails that decompose it is max {k, 1}. **6**

c) Write a note on hypercube $Q_k$. **6**

**3.** a) State and prove Cayley's formula for trees. **8**

b) Show that the center of a tree is a vertex or an edge. **4**

c) Show that in a strong digraph, every vertex is the root of an out-tree. **4**

4. a) Explain Dijkstra's Algorithm and prove that given a graph G. and a vertex $u \in V(G)$, Dijkstra's Algorithm computes $d(u, z)$ for every $z \in V(G)$. 10

b) Define caterpillar and show that a tree is caterpillar if and only if it does not contain the tree Y. 6

5. a) Prove that a matching M in a graph G is a maximum matching in G if and only if G has no M-augmenting path. 6

b) State Hall's theorem and deduce that for K > O, every k-regular bipartite graph has a perfect matching. 6

c) Define dominating set and domination number and show that a set of vertices in a graph is an independent dominating set if and only if it is a maximal independent set. 4

6. a) Write a note on Harary graphs. 4

b) If G is a simple graph, then show that $k(G) \leq k'(G) \leq \delta(G)$. 6

c) A graph is 2-connected if and only if it has an ear decomposition. Furthermore, every cycle in a 2-connected graph is the initial cycle in some ear decomposition. 6

7. a) State and prove Menger's theorem. 12

b) Explain Ford-Fulkerson labeling algrithm. 4

8. a) Define uniform Matroid and partition Matroid. Give an example of each. 4

b) State and prove Matroid intersection theorem. 12

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Seat Number

MT-104
# Ordinary and Partial Differential Equations
## (New) (111104)

P. Pages : 2

Time : Three Hours Max. Marks : 80

Instructions to Candidates :
1. Do not write anything on question paper except Seat No.
2. Graph or diagram should be drawn with the black ink pen being used for writing paper or black HB pencil.
3. Students should note, no supplement will be provided.
4. Attempt **any five** questions of the following.
5. All questions carry equal marks.
6. Figures to the right indicate full marks.

1. a) Using the method of undetermined coefficients, solve $(D^2 + 2D + 1)y = x - e^x$ **8**

   b) Apply the method of variation of parameters to solve $\frac{d2y}{dx^2} - y = \frac{2}{1+e^x}$ **8**

2. a) Solve $\left(\frac{y^2z}{x}\right)p + xzq = y^2$ **4**

   b) Solve $\left\{\frac{b-c}{a}\right\} yzp + \left\{\frac{c-a}{b}\right\} z\,xq = \left\{\frac{a-b}{c}\right\} xy$ **6**

   c) Find the equation of integral surface of the linear partial differential equation $x(y^2 + z)p - y(x^2 + z)q = (x^2 - y^2)z$ which contains the straight line $x + y = 0;\ z = 1$. **6**

3. a) Find a complete integral of $z = px + qy + p^2 + q^2$ by using Charpit's method. **6**

   b) Solve $pq = k$, where k is a constant. **4**

c) Find the complete integral of $z^2p^2y + 6zpxy + 2zqx^2 + 4x^2y = 0$ **6**

**4.** a) Explain Jacobi's method used for solving partial differential equations involving three independent variables. **10**

b) Find a complete integral of $p_1^3 + p_2^2 + p_3 = 1$, where **6**

$p_1 = \frac{\partial z}{\partial x_1}; p_2 = \frac{\partial z}{\partial x_2}; p_3 = \frac{\partial z}{\partial x_3}$.

**5.** a) Explain the method of finding the complementary function (C.F) of the linear homogeneous partial differential equation with constant coefficients, $F(D,D')z = f(x,y)$. **10**

b) Solve $(D^2 + 3DD' + 2D'^2)z = x + y$. **6**

**6.** a) Show that the equation $\frac{\partial^2 y}{\partial t^2} + 2k\frac{\partial y}{\partial t} = c^2\left(\frac{\partial^2 y}{\partial x^2}\right)$ possesses solution of the form $\sum_{r=0}^{\infty} C_{re}^{-kt} \cdot \cos(w_r t + \delta r) \times \cos(\alpha_r x + \epsilon_r)$ where $C_r, \alpha_r, \delta_r, \epsilon_r$ are constants and $w_r^2 = \alpha_r^2 c^2 - k^2$. **8**

b) Solve $(D^2 + DD' + D' - 1)z = \sin(x + 2y)$ **8**

**7.** a) Write down the canonical form of one - dimensional wave equation $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} = 0$. **8**

b) Write down working rule for reducing a parabolic equation to its canonical form. **8**

**8.** a) Explain Monge's method of integrating $R_r + S_s + T_t = V$ where R, S, T, V are functions of x, y, z, p and q. **10**

b) Form a partial differential equation by eliminating h and k from the equation from the equation $(x - h)^2 + (y - k)^2 + z^2 \lambda^2$. **6**

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Seat Number

* 3 0 4 7 *

# MT-104
# Differential Equations (Old)
# (104)

**P. Pages : 2**

**Time : Three Hours** **Max. Marks : 80**

Instructions to Candidates :
1. Do not write anything on question paper except Seat No.
2. Graph or diagram should be drawn with the black ink pen being used for writing paper or black HB pencil.
3. Students should note, no supplement will be provided.
4. Attempt **any five** questions.
5. Figures to the right indicate full marks.

**1.** a) Solve : $(D^2-2D-3)y=2e^x-10\sin x$ **4**

b) Solve : $(x^2D^2-6xD+10)y=3x^4+6x^3$ **6**

c) Solve : $(D^3-6D^2+11D-6)y=e^x$ using the method of variation of parameters. **6**

**2.** a) Find the series solution of $y''+(x-1)^2y'-4(x-1)y=0$ about x = 1. **6**

b) Using the Frobenius method solve the differential equation $x^2y''+3xy'+(1-2x)y=0$ **10**

**3.** a) Solve : $x^2y''+xy'+(x^2-1)y=0$ **8**

b) Solve : $(1-x^2)y''-2xy'+2y=0$ **8**

**4.** a) Prove that $P_n(x)$ is the coefficient of $t^n$ in the expansion of $(1-2xt+t^2)^{-\frac{1}{2}}$ where $|x|\le 1,\ |t|<1$ **8**

b) Prove that i) $(2n+1)P_n(x) = P'_{n+1}(x) - P'_{n-1}(x)$ **8**

ii) $P'_n(x) - xP'_{n-1}(x) = nP_{n-1}(x)$

**5.** a) Prove that i) $J_n(-x) = (-1)^n J_n(x)$ **8**

ii) $\cos(x\sin\phi) = J_0(x) + 2\cos 2\phi\, J_2(x) + .....$

b) Prove that $\exp\left\{\frac{1}{2}x\left(t-\frac{1}{t}\right)\right\} = \sum_{-\infty}^{\infty} t^n J_n(x)$ **8**

**6.** a) Show that $z = \frac{1}{x}\phi(y-x) + \phi'(y-x)$ is a solution of p.d.e. **4**

$x^2[Z_{xx} - Z_{yy}] = 2Z$.

b) Solve : $\gamma + s - 2t - p - 2q = 0$. **4**

c) If u, v, w are prescribed functions of x and y then show that $Z = f(u) + g(v) + w$ is a solution of $Pp + Qq + Rr + Ss = W$ where P, Q, R, S, T, W are known functions of x and y. **8**

**7.** a) If $\beta r D' + \gamma_r$ is a factor of $F(D, D')$ and $\phi_r(\xi)$ is an arbitrary function of single variable $\xi$ and $\beta_r \neq 0$ then $\exp\left[\frac{-\gamma_r}{\beta_r}y\right]\phi_r(\beta_r x)$ is a solution of $F(D,D') = 0$. **8**

b) Reduce the equation $\frac{\partial^2 z}{\partial x^2} + x^2\frac{\partial^2 z}{\partial y^2} = 0$ to Canonical form. **4**

c) Find a particular integral of the equation $(D^2 - D')Z = e^{2x+y}$. **4**

**8.** a) Show that $\frac{\partial^2 z}{\partial x^2} = \frac{1}{k}\frac{\partial z}{\partial t}$ has a solution of the form. **8**

$z(x,t) = c_n e^{-n^2 kt}\cos(nx + \epsilon_n)$ where $c_n$ is constant.

b) Reduce the equation $Rr + Ss + Tt + f(x, y, z, r, q) = 0$ to Canonical form. **8**

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Seat Number

*3048*

# MT-304
# Lattice Theory
# (341)

**P. Pages : 3**

**Time : Three Hours** **Max. Marks : 80**

Instructions to Candidates :
1. Do not write anything on question paper except Seat No.
2. Graph or diagram should be drawn with the black ink pen being used for writing paper or black HB pencil.
3. Students should note, no supplement will be provided.
4. Attempt **any five** questions.
5. Figures to the right indicate full marks.

**1.** a) Define a poset. Let A be the set of all normal subgroups of a group G. For $x, y \in A$ define $x \le y$ if x is a normal subgroup of y. Is $(A, \le)$ a poset? Is it lattice? Justify your answer. **6**

b) Let $(P, \le)$ be a finite poset. Prove that in P, $a \le b$ if and only if a = b or there exists a finite sequence of elements $x_0, x_1, \ldots\ldots x_{n-1}$ such that $x_0 = a, x_n = b$ and $x_i -< x_{i+1}$, for $0 \le i < n-1$. **6**

c) Construct an example of a poset which is not a lattice. **4**

**2.** a) Let I be an ideal and let D be a dual ideal in a lattice L. If $I \cap D \neq \phi$, show that $I \cap D$ is a convex sublattice and every convex sublattice can be expressed in this form in one and only one way. **8**

b) Show that the following inequalities hold in any lattice : **8**

i) $(x \wedge y) \vee (x \wedge z) \le x \wedge (y \vee z)$

ii) $x \vee (y \wedge z) \le (x \vee y) \wedge (x \vee z)$

iii) $(x \wedge y) \vee (y \wedge z) \vee (z \wedge x) \le (x \vee y) \wedge (y \vee z) \wedge (z \vee x)$

iv) $(x \wedge y) \vee (x \wedge z) \le x \wedge (y \vee (x \wedge z))$

3. a) Let $L, L_1, K, K_1$ be lattices such that $L \cong L_1$ and $K \cong K_1$, prove that $L \times K \cong L_1 \times K_1 \cong K_1 \times L_1$. **8**

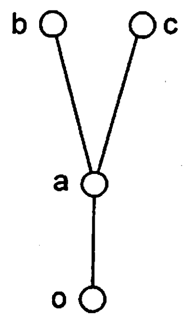
b) Define an isotone map. Prove that a meet homomorphism, join homomorphism and lattice homomorphism, are all isotone. **6**

c) Give an example of an ideal in a lattice which is not prime. **2**

4. a) State and prove De-Morgan identities in a bounded distributive lattice. **6**

b) Prove that every maximal chain C of a finite distributive lattice L, is of length $|J(L)|$. **6**

c) Are the following Lattices? Justify. **4**

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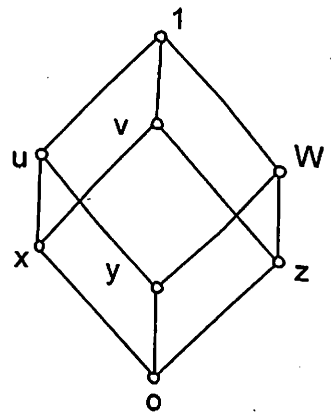


(ii)

5. a) Prove that a lattice is distributive if and only if it is isomorphic to ring of sets. **8**

b) Prove that a finite distributive lattice is a stone lattice if and only if it is the direct product of finite distributive lattices. **8**

6. a) Prove that a modular lattice is distributive if and only if it does not contain a diamond. **6**

b) Show that in a finite lattice every element is the join of join - irreducible elements. **6**

c) Let L be any arbitrary lattice. Show that C(L), the lattice of congruence relations on L, is distributive. **4**

7. a) Let L be a distributive lattice with $0 \neq 1$. Prove that L is a Boolean lattice if and only if P (L) is unbounded. 8

b) Let L be a pseudo complemented meet semi lattice. Let $S(L) = \{a^* / a \in L\}$. Prove that the partial ordering of L partially orders S(L) and makes S(L) into a Boolean lattice. For $a, b \in S(L)$, we have $a \wedge b \in S(L)$ and $a \vee b = (a^* \wedge b^*)^*$. 8

8. a) Prove that every principal congruence relation is compact. 8

b) Show that every neutral element is standard and that every standard element is distributive. 4

c) Prove or disprove : A modular lattice is distributive. 4

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Seat Number

MT-204

# Mathematical Methods

## (New) (111204)

P. Pages : 2

Time : Three Hours

Max. Marks : 80

Instructions to Candidates :

1. Do not write anything on question paper except Seat No.
2. Graph or diagram should be drawn with the black ink pen being used for writing paper or black HB pencil.
3. Students should note, no supplement will be provided.
4. Attempt **any five** questions.
5. Figures to the right indicate full marks.

1. a) Derive Laplace Equation in cylindrical co-ordinates. 7

   b) Explain the terms with suitable examples. 9
   i) Linear bvp
   ii) Linear combination
   iii) Uniform convergence of a series.

2. a) State and prove principle of superposition. 8

   b) Solve the following p de: 8
   i) $U_{xx} = 6xy,\ u(0,y) = y,\ u_x(1,y) = 0$
   ii) $U_{xx} = 2x,\ u(0,y) = 0,\ u(x,0) = x^2$

3. a) Solve the following bvp: 8

$$y_{tt}(x,t) = a^2\, y_{xx}(x,t),$$
$$y(0,t) = (c,t) = 0,$$
$$y_t(x,0) = 0$$
$$y(x,0) = \begin{cases} x, & 0 < x < Cl_2 \\ c - x, & \frac{c}{2} < x < C \end{cases}$$

   b) Solve the following bvp:- 8

$$y_{tt}(x,t) = a^2 y_{xx}(x,t)$$
$$y_x(0,t) = y_x(c,t) = 0,$$
$$y(x,o) = bx,\ y_t(x,0) = 0$$

4. a) Solve the following bvp. **8**
$u_t(x,t) = k u_{xx}(x,t)$,
Subject to the conditions:
$u(0,t) = 0, u_x(\pi,t) = u, u(x,0) = u_o \sin x$

b) Solve the following bvp. **8**
$u_t(x,t) = k\, u_{xx}(x,t)$
$u(0,t) = u,\ u(1,t) = 1,\ u(x,0) = 0$

5. a) Explain the following terms: **9**
i) Inner product of two vectors.
ii) Orthonormal sets of vectors.
iii) Generalized Fourier series.

b) If on orthonormal & set is closed, show that it is complete. **7**

6. a) Show that eigen values of S-L problem are real. **7**

b) Solve the following SL problem **5**
$x'' + \lambda x = 0,\ x'(0),\ x(c) = 0$

c) Show that the set $\left\{\frac{1}{\sqrt{C}} \sin \frac{n\pi x}{c}\right\}$ (n =1,2,..........) is orthonormal on (o,c) **4**

7. a) Solve the Dirichlet problem: **8**
$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$, subject to the conditions.
$u(0,y) = u(x,0) = u(a,y) = 0,\ u(x,b) = u_o$.

b) Solve : $\rho^2 u_{ss} + \rho u_\rho + u_{\phi\phi} = 0\ (1 < \rho < 2,\ 0 < \phi < \pi)$ subject to the conditions. **8**
$u(\rho,0) = u(\rho,\pi) = 0$,
$u(l,\phi) = 0,\ u(2,\phi) = u_o$

8. a) Show that $\lim_{x \to \infty} J_n(x) = 0$ **8**

b) Write an explanatory note on the following. **8**
i) Bessel functions of order o
ii) Recurrence relations for $J_n(x)$

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Seat Number

* 3 0 5 0 *

# MT-204
# Mathematical Methods
## (Old) (204)

P. Pages : 2

Time : Three Hours

Max. Marks : 80

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Instructions to Candidates :

1. Do not write anything on question paper except Seat No.
2. Graph or diagram should be drawn with the black ink pen being used for writing paper or black HB pencil.
3. Students should note, no supplement will be provided.
4. Attempt **any five** questions of the following.
5. All questions carry equal marks.
6. Figures to the right indicates full marks.

**1.** a) Derive heat equation and obtain its separated solution. **6**

b) Derive one - dimensional wave equation. **6**

c) Show that if an operator L has the two properties **4**

$L(u_1 + u_2) = L(u_1) + L(u_2)$

$L(c_1 u_1) = c_1 L(u_1)$

For all functions $u_1, u_2$ in some space and every constant $C_1$, then L is Linear.

**2.** a) Show that an orthonormal set $\{\phi_n(x)\}$ is closed in a given function space if it is complete in that space. **8**

b) Solve $u_t(x,t) = K u_{xx}(x,t), 0 < x < \Pi, t > 0$ with **8**

$u(0,t) = 0, u(\Pi,t) = 0$ and $u(x,0) = f(x)$.

**3.** a) Define orthonormal set, show that the set **6**

$\{\phi_n(x)\}$ $n = 0,1,2,$...............with $\phi_0(x) = \frac{1}{\sqrt{\Pi}}$ and

$\phi_n(x) = \sqrt{\frac{2}{\Pi}}.\cos nx$ $(n = 1, 2,$............$)$ is orthonormal on the interval $0 < x < \Pi$.

b) Let f and g denote any two functions in the space Cp (a,b). Show that if either function has zero norm, then (f,g) = 0. **6**

c) Prove that if f and g are functions in the space Cp (a,b), then $\|fg+g\| \leq \|f\| + \|g\|$. **4**

4. a) Prove that Bessel's inequality and further more $\lim_{n\to\infty} C_n = 0$. **8**

b) Find the eigen values and normalized eigen functions of $X'' + \lambda X = 0, X(0) = 0, X'(1) = 0$. **8**

5. a) Explain orthogonality of an eigen functions. **6**

b) Define closed orthonormal set and complete orthonormal set. **2**

c) Solve $X'' + \lambda X = 0$ with boundary conditions $X(0) = 0, hX(1) + X'(1) = 0, (h > 0)$. **8**

6. a) State and prove existence and uniqueness theorem for difference equation. **8**

b) Apply matrix method to solve. **8**

$X_{h+1} - 3X_h + 2X_{h-1} = 0 \qquad h = 1, 2, \ldots\ldots\ldots$

$X_0 = 0, X_1 = 1$

7. a) Obtain the Particular solution of the difference equation by method of undetermined coefficient **8**

$y_n + 2 - 2y_{n+1} + y_n = 2^n (n-1) 5 + 3n + n^2 + \sin 2n\Pi$.

b) Explain matrix method for solving a system of Linear difference equation. **8**

8. a) Solve the difference equation. **6**

$U_{x+2} - 4u_x = 9x^2$.

b) Solve the following system of equations. **7**

$u_{n+1} - v_n = 2(n+1)$

$v_{n+1} - u_n = -2(n+1)$

c) Explain the difference equation with suitable example. **3**

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Seat Number

*30051*

# MT-405
# Applied Numerical Methods
## (442)

P. Pages : 2

Time : Three Hours **Max. Marks : 80**

Instructions to Candidates :
1. Do not write anything on question paper except Seat No.
2. Graph or diagram should be drawn with the black ink pen being used for writing paper or black HB pencil.
3. Students should note, no supplement will be provided.
4. Attempt **any five** questions.
5. Figures to the right indicate full marks.
6. Scientific calculator is allowed.

**1.** a) Explain DO little Algorithm. **8**

b) Solve the following system Crout's method. **8**
x+y+z = 3, 2x-y + 3z = 16, 3x+y-z = -3.

**2.** a) Solve the following system by Jacobi's literation method in error format : **8**
3x+4y+15z = 54.8, x+12 y+3z = 39.66, 10x+y-2z = 7.74.

b) Solve the following system by relaxation method : **8**
10x-2y-2z = 6, -x-10y-2z = 7, -x-y+10z=8.

**3.** a) Derive Trapezoidal rule and hence find error. **8**

b) Evaluate $\int_{y=1}^{1.5}\int_{x=1}^{2} \frac{dxdy}{(x+y)}$ by using Simpson's $\frac{1}{3}$ rule. **8**

**4.** a) Derive expression for $\frac{d^2y}{dx^2}$ using backward differences. **5**

b) Find first, second and third derivatives of f (x) at x = 1.5 if : **6**

| X: | 1.5 | 2.0 | 2.5 | 3.0 | 3.5 | 4.0 |
|---|---|---|---|---|---|---|
| F(x) | 3.375 | 7 | 13.625 | 24 | 38.875 | 59 |

c) Explain Romberg's method. **4**

**5.** a) Use Picards method to solve $\frac{dy}{dx}=3x+y^2, y(0)=1$, to find y (0.1) and y (0.2). **8**

b) Solve $\frac{dy}{dx}=x^2-y, y_{(0)}=1$ find y (0.1), y (0.2) by using Fourths order R-K method. **8**

**6.** a) Explain how to solve a hyperbolic p.d.e by using finite difference scheme. **8**

b) Solve $\nabla^2 u=8x^2y^2$ with square mesh given u=0 on the four boundaries dividing the square into 16 sub squares of length one unit. **8**

**7.** a) Discuss Leibman's interation method. **8**

b) Given $u_t = 25u_{xx}$, u (0,t) = u (10, t) = 0, $u(x,0)=\frac{x}{25}(10-x)$, choosing h = 1 and k suitably, find $u_{ij}$ for $0 \leq i \leq 9$, $1 \leq j \leq 4$ by using Crank - Nicholson method. **8**

**8.** a) Discuss the Secant method to solve a non - linear B.V.P. **8**

b) Solve the B.V.P. $u''=xu, u(0)+u^1(0)=1, u(1)=1$ with $h=\frac{1}{3}$ by using second order method. **8**

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Seat Number

ANY ONE OF THE FOLLOWING : MT-106
# Programming in C++
## (New) (111152)

P. Pages : 2

Time : Three Hours

Max. Marks : 80

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Instructions to Candidates :
1. Do not write anything on question paper except Seat No.
2. Graph or diagram should be drawn with the black ink pen being used for writing paper or black HB pencil.
3. Students should note, no supplement will be provided.
4. Attempt **any five** questions.
5. Figures to the right indicate full marks.

**1.** a) Write a program to calculate area and perimeter of a rectangle. **4**

b) Explain the following terms : **12**

i) Scope of a variable
ii) input operator
iii) ASCI code
iv) Identifier
v) Key word
vi) Object

**2.** a) Write a short note on the following : **12**

i) Header files
ii) Type conversion
iii) Conditional operator
iv) Logical operator
v) Composite assignment
vi) Integer Arithmetic

b) Convert the following into C++ statement : **4**

i) $\cos^{-1}(x^3 - y^3) + (x^2 + 2xy + y^4)^{3/2}$

ii) $\log_{10}\sqrt{x^2 + y^2} + e^{(x^2+y^2)}$

**3.** a) Describe else-if structure and write a program using it. **5**

b) Write a program to calculate : **6**

$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + .....$ correct upto 4 decimals.

c) Write a program to evaluate : **5**

$$f(x) = |x^4 + y^4|, -4 < x \le -2$$
$$= |x^4 - y^4|^3, -2 < x \le 0$$
$$= \sqrt{x^2 + y^2}, 0 < x \le 2$$
$$= 0 \quad , \quad x > 2$$

**4.** a) Discuss the structure and usage of for loops. **6**

b) Use while loop to evaluate **6**

$$sum = \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{5^2} + \ldots\ldots \text{ upto input n.}$$

c) Write a note on in built functions. **4**

**5.** a) Write a short note on functions. What are the differences between call by value and call by reference. **5**

b) Write a test function for combination function. **6**

c) Write a program to find the sum and average of given set of 1000 numbers. **5**

**6.** a) How to initialize and print two dimensional array in sequential order. **5**

b) Write a program to evaluate product of two matrices. **6**

c) Write a program to compute : $\sigma = \sqrt{\frac{1}{n}\sum(x_i - \bar{x})^2}$ **5**

**7.** a) Explain Newton-Raphson method and write a program. **8**

b) Fit a polynomial using Laggrange's method. **8**

| x : | 2 | 4 | 6 | 8 |
|---|---|---|---|---|
| p(x) : | 2 | 14 | 34 | 62 |

**8.** a) Write a note on the following : **12**

i) Go to statement  ii) Exit & break
iii) Void function  iv) Continue

b) Write a program using bubble sort algorithm. **4**

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Seat Number

* 3 0 5 5 *

**ANY ONE OF THE FOLLOWING : MT-106**

# Programming in C++ with Numerical Method (Old) (106)

**P. Pages : 2**

**Time : Three Hours** **Max. Marks : 80**

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Instructions to Candidates :
1. Do not write anything on question paper except Seat No.
2. Graph or diagram should be drawn with the black ink pen being used for writing paper or black HB pencil.
3. Students should note, no supplement will be provided.
4. Attempt **any five** questions.
5. Figures to the right indicate full marks.

**1.** a) Explain different types of arithmetic operators. **6**

b) Write a program to convert the temperature in Celsius to Fahrenheit by using $C=\frac{5}{9}(F-32)$. **6**

c) Write a program to find the volume and surface area of a cube of sides $\ell$, b and h. **4**

**2.** a) Explain C in and C out with suitable examples. **6**

b) Discuss the following terms with suitable examples. **10**

i) Floating point overflow and underflow.

ii) Logical operators

iii) Scope of a variable

iv) Compound assignment operators

v) Object constant and variable.

**3.** a) Explain the following structures and write a simple program **8**

i) Else - if ii) Switch

b) Write a program to evaluate the following expression. **4**

$1-\frac{1}{2^3}+\frac{1}{3^3}-\frac{1}{4^3}+---\frac{1}{x^3}$

c) Write an equivalent expression in C++ for the following. **4**

i) $|x^{3/2} - y^{3/2}| + \sqrt{x^3 + y^3} + \log|x^v + y^v| + e\sqrt{x^v - y^v}$

ii) $\left[\left|\frac{x^v + y^v}{x^v - y^v}\right|^v + \left|\sqrt{(x^v + y^v)^v}\right|\right] / \sin^{-1}(x^v + y^v)$.

4. a) Discuss the salient features of a while loop. **6**

b) Write a program to evaluate. **6**

$e^{-x} = 1 - x + \frac{x^v}{2!} - \frac{x^3}{3!} + \frac{x^4}{4!} + - - - - - -$ (By using do while).

c) Discuss the importance of the following term. **4**

i) Continue ii) Return

5. a) What is an array? Explain how to declare and initialize a three - dimensional array. **6**

b) Write a program to find the sum of two matrices A and B (Order 3). **6**

c) What is <C math> Explain. **4**

6. a) What is function pass by value. How does it differ from function pass by reference. **6**

b) Write and test permutation function. **6**

c) Write a program to find the first 100 terms of Fibonacci sequence. **4**

7. a) Find a real root of $3x = \sqrt{1 + \sin x}$ by Bisection method. **6**

b) Determine F (x) as a polynomial in x for the following data. **5**

| X: | 0 | 2 | 3 | 6 |
|---|---|---|---|---|
| F(x): | 648 | 704 | 729 | 792 |

c) Derive Newton's forward difference interpolation formula. **5**

8. a) Apply Hermit's interpolation formula to find the values of sin (1.05) from the following data : **8**

| X: | 1.00 | 1.10 |
|---|---|---|
| Sinx | 0.88147 | 0.89121. |
| Cosx | 0.54030 | 0.45360 |

b) Write a program to find a real root of F (x) = 0 by Bisection method. **8**

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Seat Number

* 3 0 5 7 *

**ANY ONE OF THE FOLLOWING : MT-205**

# Analytical Number Theory (New)

**(111251)**

**P. Pages : 2**

**Time : Three Hours** **Max. Marks : 80**

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Instructions to Candidates :

1. Do not write anything on question paper except Seat No.
2. Graph or diagram should be drawn with the black ink pen being used for writing paper or black HB pencil.
3. Students should note, no supplement will be provided.
4. Attempt **any five** questions.
5. Figures to the right indicates full marks.

**1.** a) If $(a,b)=1$ then show that $(a+b, a^2-ab+b^2)=1$ or 3. **6**

b) If $a>1$ and m, n are positive integers then show that $(a^m-1, a^n-1)=a^{(m,n)}-1$ **6**

c) Prove that there are infinitely many prime numbers. **4**

**2.** a) If $n\geq 1$ then prove that $\sum_{d/n}\phi(d)=n$. **6**

b) If $n\geq 1$ then show that $\phi(n)=n\prod_{p/n}\left(1-\frac{1}{p}\right)$. **6**

c) Find $\mu(7007)$ and $\sigma_2(45)$. **4**

**3.** a) Define multiplicative function. If f and g are multiplicative then prove that their Dirichlet product $f*g$ is multiplicative. **6**

b) State and prove the mobius inversion formula. **6**

c) With usual notations, prove that $\alpha^{-1}(n)=1|\mu(n)|$, where $n\in N$. **4**

4. a) Let $(a,m)=d$, prove that the linear congruence $ax\equiv b(\text{mod}\,m)$ has solutions iff $d/b$. **6**

b) Prove that $a\equiv b(\text{mod}\,m)$ if and only if a and b give the same remainder when divided by m. **6**

c) Prove that if $a\equiv b(\text{mod}\,m)$ then $a^2\equiv b^2(\text{mod}\,m)$. Is the converse true? Justify your answer. **4**

5. a) State and Prove the Chinese Remainder Theorem. **6**

b) Solve the following system of linear congruences. **6**
$x\equiv 1(\text{mod}\,3)$, $x\equiv 2(\text{mod}\,4)$, $x\equiv 3(\text{mod}\,5)$

c) For any prime $p\geq 5$, prove that **4**
$$\sum_{k=1}^{p-1}\frac{(p-1)!}{K}\equiv 0(\text{mod}\,p^2)$$

6. a) State and prove Gauss Lemma. **8**

b) For every odd prime p, show that **4**
$$(2/p)=\begin{cases}1 \text{ if } p\equiv\pm 1(\text{mod}\,8)\\ -1 \text{ if } p\equiv\pm 3(\text{mod}\,8)\end{cases}$$

c) Find (219/383) **4**

7. a) If p and q are distinct odd primes then prove that **8**
$(p/q)(q/p)=(-1)^{(p-1)(q-1)/4}$

b) If $(a,m)=1$ and $f=\exp_m(a)$ then prove that **8**
$$\exp_m(a^k)=\frac{\exp_m(a)}{(k,f)}$$

8. a) Define primitive root mod m. Let (a, m) = 1. Prove that a is a primitive root mod m if and only if the numbers $a,a^2,\ldots,a^{(m)}$ form RRS (mod m). **8**

b) Let g be a primitive root modulo p such that $g^{p-1}\not\equiv 1(\text{mod}\,p^2)$. **8**

Then prove that for every $\alpha\geq 2$ we have $g^{\phi(p^{\alpha-1})}\not\equiv 1(\text{mod}\,p^{\alpha})$.

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Seat Number

* 3 0 5 9 *

## ANY ONE OF THE FOLLOWING : MT-205
# Solid Mechanics
## (Old) (251)

P. Pages : 2

Time : Three Hours Max. Marks : 80

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Instructions to Candidates :
1. Do not write anything on question paper except Seat No.
2. Graph or diagram should be drawn with the black ink pen being used for writing paper or black HB pencil.
3. Students should note, no supplement will be provided.
4. Attempt **any five** questions.
5. Figures to the right indicate full marks.
6. Scientific calculator is allowed.
7. Usual notations are employed.

1. a) Explain the following terms : **12**
   i) Deflection curve
   ii) Pure bending of symmetric beams
   iii) Mohr's circle

   b) Find the clongation of a bar of cross-section A and weight $W_o$ per unit length under action of it's self weight. **4**

2. a) Derive the equations of equilibrium in differential form. **8**

   b) Given $\tau_{xx} = 6x^2 + 3xy + 20pa$, $\tau_{yy} = 2xz + 10pa$, $\tau_{zz} = 4x^2 + 3y + 10yz$ pa, What must be the body force for equilibrium? What is the body force at (3,2,4)m? **8**

3. a) Derive 3-D Hooke's law. **6**

   b) Show that $\tau_{xy} = \tau_{yx}$. **6**

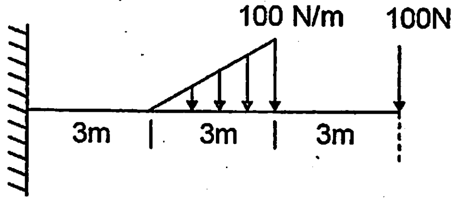
   c) Write a short note on stresses and strains. **4**

4\. a) Derive expressions for $\epsilon'_{xx}$, $\epsilon'_{yy}$ and $\epsilon'_{xy}$ interms of $\epsilon_{xx}$, $\epsilon_{yy}$ and $\epsilon_{xy}$. **8**

b) If $\tau_{xx} = 4000pa$, $\tau_{yy} = 1000pa$, $\tau_{xy} = -500pa$ find the principal stresses and principal strains if E = 200Gpa and v = 0.2. **8**

5\. a) Show that $\bar{\tau}_{yx} = \frac{V_y Q_z}{bI_{zz}}$ **8**

b) Find the shear and B. M equations for the following beam. **8**



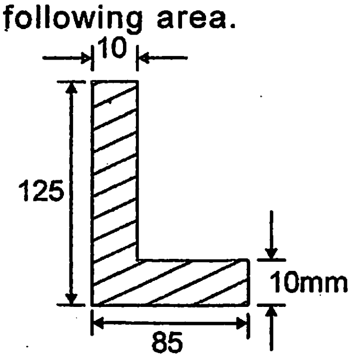

6\. a) Define Neutral surface and radius of curvature of a beam and hence show that $\epsilon_{xx} = -Y/R$. **8**

b) Show that $\frac{d^4M_z}{dx^4} = W_y(x)$ and obtain the solution. **8**

7\. a) Find expressions for $I_{xx}$ and $I_{yy}$ for a semi-circle of diameter D. **6**

b) State and prove parallel axes theorem. **4**

c) Find $I_{xx}$ and $I_{yy}$ for the following area. **6**




8\. a) Find the expression for strength of a solid shaft. **4**

b) Define torsional stress and strain. **4**

c) Show that $\frac{q}{r} = \frac{fs}{R} = \frac{G\theta}{\ell}$ **8**

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Seat Number

*3060*

# ANY ONE OF THE FOLLOWING : MT-206
## Number Theory
## (Old) (252)

**P. Pages : 2**

**Time : Three Hours** **Max. Marks : 80**

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Instructions to Candidates :
1. Do not write anything on question paper except Seat No.
2. Graph or diagram should be drawn with the black ink pen being used for writing paper or black HB pencil.
3. Students should note, no supplement will be provided.
4. Attempt **any five** questions.
5. Figures to the right indicate full marks.

**1.** a) Prove that any natural number $n > 1$ can be written as a product of finite number of primes and further the primes occuring in the product representation are unique upto the order of primes occuring in the product. **6**

b) Show that for any integer a, $(2a+1, 9a+4) = 1$. **5**

c) If $d|n$, then show that $3^d - 1$ divides $3^n - 1$. **5**

**2.** a) If $n \geq 1$, then prove that $\phi(n) = \sum_{d|n} \mu(d) \cdot \frac{n}{d}$ **5**

b) Find : **5**

i) $\sigma_2(72)$ ii) $\mu(2431)$ iii) $\phi^{-1}(3211)$

c) Prove that : **6**

i) $\phi$ is multiplicative function.

ii) $\phi(n)$ is even, for all $n \geq 3$.

**3.** a) State and prove Mobius inversion formula. **6**

b) Let f, g be arithmetic functions. If $g, f*g$ are multiplicative functions, then prove that f is multiplicative. **6**

c) Prove that d(n) is odd if and only if n is a square. **4**

4. a) if $a \equiv b \pmod{m}$, then prove that $a^n \equiv b^n \pmod{m}$, $\forall\, n \in \mathbb{N}$. Is the converse true ? Justify. 6

b) Find the remainder when $(23)^{23} \times (23)!$ is divided by 29. 6

c) If P is a prime, then show that $(p-2)!-1$ is divisible by p. 4

5. a) Prove that for any prime p, all the coefficient of the polynomial $f(x) = (x-1)(x-2)....(x-p+1) - x^{p-1} + 1$ are divisible by p. 6

b) Solve $x \equiv 2 \pmod 3$ 5

$x \equiv 3 \pmod 5$

$x \equiv 2 \pmod 7$

c) If p, q are distinct primes, then show that $(p^q + q^p) \equiv (p+q) \pmod{pq}$. 5

6. a) State and prove Gauss lemma. 6

b) let p be an odd prime. Prove that ${}_5R_p$ if $p \equiv \pm 1 \pmod{10}$ and ${}_5\overline{R}_p$ if $p \equiv \pm 3 \pmod{10}$. 6

c) If p is an odd prime and $a \equiv b \pmod{p}$, then show that $(a|p) = (b|p)$. 4

7. a) let a, b be odd positive integers and (a, b) = 1. Prove that 6

$$\sum_{r=1}^{\frac{b-1}{2}} \left[\frac{ra}{b}\right] + \sum_{s=1}^{\frac{a-1}{2}} \left[\frac{sb}{a}\right] = \frac{a-1}{2} \cdot \frac{b-1}{2}$$

b) Let (a, m) = 1. Prove that a is a primitive root mod m if and only if $\{a, a^2, \ldots, a^{\phi(m)}\}$ forms a RRS mod m. 6

c) True or false ? Justify, 68 has a primitive root. 4

8. a) If (a, m) = (b, m) = 1 and $(\exp_m(a), \exp_m(b)) = 1$, then prove that $\exp_m(ab) = \exp_m(a) \cdot \exp_m(b)$. 4

b) Show that $2^n$ has no primitive root, for all $n \geq 3$. 6

c) Find a primitive root of 242. 3

d) Find (187|2013). 3

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Seat Number

* 3 0 6 1 *

# MT - 406
# Algebraic Topology
## (451)

**P. Pages : 3**

**Time : Three Hours** **Max. Marks : 80**

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Instructions to Candidates :

1. Do not write anything on question paper except Seat No.
2. Graph or diagram should be drawn with the black ink pen being used for writing paper or black HB pencil.
3. Students should note, no supplement will be provided.
4. Attempt **any five** questions.
5. Figures to the right indicates full marks.

**1.** a) Let X be a topological space and $A \subseteq X$. Prove that the relation of being homotopic relative to A is an equivalence relation. **7**

b) If $f_1, f_2 : X \to Y$, $g_1, g_2 : Y \to Z$ are continuous mappings such that $f_1 \simeq f_2$ and $g_1 \simeq g_2$ then show that $g_1 o f_1 \simeq g_2 o f_2$. **5**

c) Show that [0, 1] is a contractible space. **4**

**2.** a) Let f be a path in a topological space X whose initial point is x & terminal point is y. Show that $f * \bar{f} \sim \epsilon_x$ and $\bar{f} * f \sim \epsilon_y$, where $\epsilon_x$ and $\epsilon_y$ are null paths in X. **8**

b) Show that IR and {0} are of the same homotopy type. **4**

c) Let X be a topological space and Y a contractible space. If $f : X \to Y$ is a continuous map then show that f is homotopic to a constant map. **4**

**3.** a) Let X be a topological space and $x_0, x_1 \in X$. If h is a path in X from $x_0$ to $x_1$, then show that $\pi_1(X, x_0) \cong \pi_1(X, x_1)$. **6**

b) Show that fundamental group of a contractible space is trivial. **6**

c) Let X, Y, Z be topological spaces and $f: X \to Y, g: Y \to Z$ be continuous mappings, show that $(gf)^* = g^* f^*$. 4

4. a) Let X, Y be topological spaces, $x_0 \in X$ and $\phi, \psi: X \to Y$ be continuous mappings. If $F: \phi \simeq \psi$, then prove that $\psi^* = \Phi_f \circ \phi^*$, where f is a path in Y from $\phi(x_0)$ to $\psi(x_0)$ defined by $f(t) = F(x_0, t)$ and $\Phi_f : \pi_1(Y, \phi(x_0)) \to \pi_1(Y, \psi(x_0))$ is an isomorphism determined by f. 8

b) Prove that every strong deformation retract of a topological space X is a retract of X. Is the converse true ? Justify. 8

5. a) Let X be a topological space. Prove that every covering map $p: \tilde{X} \to X$ is open. 6

b) Let X be a topological space, $p: \tilde{X} \to X$ a covering map and $f_1, f_2 : Y \to \tilde{X}$ be two liftings of $f: Y \to X$, where Y is connected space. If there exists $y_0 \in Y$ such that $f_1(y_0) = f_2(y_0)$ then prove that $f_1 = f_2$. 6

c) If $\mathbb{Z}$ acts on $\mathbb{R}$ by n.x = n+x, then find $\mathbb{R}/\mathbb{Z}$. 4

6. a) Let $S = \{a_0, a_1, ......a_p\}$ be a geometrically independent set in $\mathbb{R}^n$. Prove that $\pi(S) = \left\{ x \in IR^n / x = \sum_{i=0}^{p} \lambda_i a_i, \sum_{i=0}^{p} \lambda_i = 1 \right\}$ and $S \subseteq \pi(S)$. 6

b) Let $S_p = (a_0, a_1, .....a_p)$ be a simplex in $\mathbb{R}^n$. Prove that 6

i) $\overline{S_p}$ = Union of all faces of $S_p$

ii) Boundary of $S_p$ is union of all proper faces of $S_p$.

c) Prove or disprove : every linearly independent set is a geometrically independent set. 4

7. a) State and prove Brouwer fixed point theorem. 8

b) Let K be a complex of dimension m. Prove that 8

$$u(k') \leq \frac{m}{m+1} u(k)$$

**8.** a) Let k be a complex and $S_m, S_{m+2}$ be simplexes in k, where $S_m$ is a face of $S_{m+2}$. Prove that **6**

$$\sum_{t_{m+1} \in k} [s_{m+2}\, t_{m+1}]\,[t_{m+1}\, s_m] = 0$$

b) Let $s_2 = (a_0, a_1, a_2)$ be a 2-simplex oriented by an order $a_0 < a_1 < a_2$. If k is a complex consisting $s_2$ and all its faces then compute homology group of k. **6**

c) Let $c_p$ is a p-boundary of a complex k then show that $\phi_p(c_p)$ is a p-boundary of L, where L is a complex. **4**

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Seat Number

* 3 0 6 3 *

# MT - 306
# Mathematical Statistics
# (351)

**P. Pages : 3**

**Time : Three Hours** **Max. Marks : 80**

Instructions to Candidates :
1. Do not write anything on question paper except Seat No.
2. Graph or diagram should be drawn with the black ink pen being used for writing paper or black HB pencil.
3. Students should note, no supplement will be provided.
4. Attempt **any five** questions.
5. Figures to the right indicate full marks.
6. Scientific calculator is allowed.
7. Mathematical and statistical tables are allowed.

**1.** a) For the following data, find $x_1$ and $x_2$ if Mode = 54 and total frequency is 94 : **5**

| Class Interval | 0 – 20 | 20 – 40 | 40 – 60 | 60 – 80 | 80 – 100 |
|---|---|---|---|---|---|
| Frequency | 10 | $x_1$ | 30 | $x_2$ | 14 |

b) Calculate H.M. of the following data : **5**
15, 250, 15.7, 157, 1.57, 105.7, 10.5, 1.06, 25.7, 0.257

c) Calculate mean deviation for the following series from mode : **6**

| Class | 0 - 10 | 10 - 20 | 20 - 30 | 30 - 40 | 40 - 50 | 50 - 60 | 60 - 70 |
|---|---|---|---|---|---|---|---|
| Frequency | 6 | 5 | 8 | 15 | 7 | 6 | 3 |

**2.** a) Find out the combined mean and S.D. from the following data : **6**

| | Series A | Series B |
|---|---|---|
| n : | 100 | 500 |
| $\bar{x}$ : | 50 | 60 |
| σ : | 10 | 11 |

b) Prove that : 5

i) $P(\overline{A} \cap B) = P(B) - P(A \cap B)$

ii) $P(\overline{A} \cap \overline{B}) = P(\overline{A})\, P(\overline{B})$ if A and B are independent.

c) Two digits are selected from 1 to 9. Find the probability that : 5

i) the sum is even

ii) both the digits are odd if the sum is even.

3. a) For a Binomial distribution, show that $\bar{x} = np$ and $\sigma^2 = npq$. 6

b) If 5% of electric bulbs are defective, use Poisson distribution to find the probability that in a sample of 100 bulbs that 5

i) none is defective

ii) at the most 3 are defective

c) Write a short note on normal frequency distribution and its properties. 5

4. a) For correlation coefficient $r_{xy}$, show that $-1 \le r \le 1$. 5

b) Calculate probable error for the following data : 6

| Sales | 50 | 50 | 55 | 60 | 65 | 65 | 65 | 60 | 60 | 50 |
|---|---|---|---|---|---|---|---|---|---|---|
| Expenditure | 11 | 13 | 14 | 16 | 16 | 15 | 15 | 14 | 13 | 13 |

c) Show that $\rho = 1 - 6\dfrac{\Sigma a^2}{n(n^2 - 1)}$ 5

5. a) Use method of least squares to derive $b_{yx}$. 6

b) Find the regression equations from the following data : 5

| x | 25 | 28 | 35 | 32 | 31 | 36 | 29 | 38 | 34 | 32 |
|---|---|---|---|---|---|---|---|---|---|---|
| y | 43 | 46 | 49 | 41 | 36 | 32 | 31 | 30 | 33 | 39 |

c) The lines of regression are : $8x - 10y + 66 = 0$, $40x - 18y = 214$ 5

If $\sigma_x = 3$, find :

i) $\bar{x}$ and $\bar{y}$ ii) $r_{xy}$ iii) $\sigma_y$

6. a) Explain large sample test for single mean. 4

b) The issue of books in a college library is as follows : **6**

| Day | Mon | Tue | Wed | Thu | Fri | Sat |
|---|---|---|---|---|---|---|
| Books | 125 | 135 | 105 | 115 | 110 | 130 |

Is the issue of books day dependent ? Take $\chi_5^2(0.01)=15.086$.

c) Out of a sample of 120 persons, 76 persons were given a new drug for preventing T.B. and out of then 24 persons were attacked by T.B. Out of those, who were not given new drug, 12 persons were not attacked. Can we say that new drug is effective in controlling T.B. Take $\chi_1^2(0.05)=3.84$. **6**

**7.** a) A sample of 900 ball bearings is found to have average weight of 12.5 g. Can we assume that the sample is coming from a population with a mean of 13g against that it is less than 13g. Take S.D. to be 1g. **6**

b) A coin is tossed 900 times and head appeared 490 times. Would you conclude that the coin is unbiased ? **5**

c) Write a short note on F-distribution. **5**

**8.** Write short note on the following. **16**

i) Types of correlation.

ii) Differences between M.D. & S.D.

iii) Type I & Type II errors.

iv) Sampling theory.

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