Khandesh College Education Society's

Moolji Jaitha College, Jalgaon

An "Autonomous College" Affiliated to
KBC North Maharashtra University, Jalgaon

KNOWLEDGE IS POWER

ESTD. 1945

SYLLABUS

Mathematics

T.Y.B. Sc.

(Semester V & VI)

Under Choice Based Credit System (CBCS)

[w. e. f. Academic Year: 2021-22]

T.Y.B.Sc. Mathematics (CBCS pattern)

Program Specific Outcomes (PSO):
Students who graduate with Mathematics will be able to

- Demonstrate the concepts involved in Real analysis, Matrix Theory, Differential equations, Algebra, Number Theory and Applied Mathematics.
- Gain proficiency in mathematical techniques of both pure and applied mathematics and will be able to apply necessary mathematical methods to a scientific problem.
- Acquire significant knowledge on various aspects related to Linear algebra, Metric spaces, Lattice theory, Integral transforms, Optimization techniques and Partial Differential equations.
- Learn to work independently as well as a team to formulate appropriate mathematical methods.
- Develop the ability to understand and practice the morality and ethics related to scientific research.
- Realize the scope of Mathematics and plan continue their education as a Post-Graduate student of Mathematics and contribute to Mathematics through their research as a doctoral student.

Learning Objectives:

- To acquaint the students with various branches of Mathematics.
- To get acquainted with the foundations and pillar level knowledge of Mathematics and use it for advanced studies in the subject.
- To develop practical skills with a sound theoretical background.
- To develop their interest in various branches of Mathematics and use it in their professional endeavours.

Exam Pattern:

- Each theory and practical course will be of 50 marks comprising of 10 marks internal and 40 marks external examination.

External Theory Examination (40 marks):

- External examination will be of two hours duration for each theory course. There shall be 4 questions each carrying equal marks (10 marks each). The tentative pattern of the question papers will be as follows:
  - Q1 (A), Q2 (A), Q3 (A) and Q4 (A), each will be of 6 marks (attempt any 2 out of 3 sub-questions).
  - Q1 (B), Q2 (B), Q3 (B) and Q4 (B), each will be of 4 marks (attempt any 1 out of 2 sub-questions).

External Practical Examination (40 marks):

- Practical examination shall be conducted by the respective department at the end of the semester. Practical examination will be of 3 hours duration and shall be conducted as per schedule. There shall be 05 marks for journal, 10 marks for *viva-voce*. Certified journal is compulsory to appear for practical examination.

Internal Theory/ Practical Examination (10 marks):

- Internal theory assessment of the student by respective teacher will be comprehensive and continuous, based on written test/assignment. The written test may comprise of both objective and subjective type questions.
- Internal practical examination shall be conducted by respective department as per schedule given.

Structure of T.Y.B.Sc. (Mathematics) Curriculum Semester V

Discipline	Course Type	Course Code	Course Title	Credits	Hours/ Week (Clock Hours)	Total Teaching hours	Marks	
							Int	Ext
DSC	Core I	MTH-351	Methods of real analysis	3	3	45	10	40
	Core II	MTH-352	Integral calculus	3	3	45	10	40
	Core III	MTH-353	Modern algebra	3	3	45	10	40
	Core IV	MTH-354	Lattice theory	3	3	45	10	40
	Core V	MTH-355	Number theory	3	3	45	10	40
	Core VI	MTH-356	Fourier transforms	3	3	45	10	40
SEC	Skill Based	MTH-350	Laplace transforms	2	2	30	10	40
DSC	Core (Practical)	MTH-357	Practical course based on MTH-351 and MTH-352	2	4 / batch	60	10	40
		MTH-358	Practical course based on MTH-353 and MTH-354	2	4 / batch	60	10	40
		MTH-359	Practical course based on MTH-355 and MTH-356	2	4 / batch	60	10	40

Structure of T.Y.B.Sc. (Mathematics) Curriculum Semester VI

Discipline	Course Type	Course Code	Course Title	Credits	Hours/ Week (Clock Hours)	Total Teaching hours	Marks	
							Int	Ext
DSC	Core I	MTH-361	Measure and integration theory	3	3	45	10	40
	Core II	MTH-362	Metric spaces	3	3	45	10	40
	Core III	MTH-363	Linear algebra	3	3	45	10	40
	Core IV	MTH-364	Partial differential equations	3	3	45	10	40
	Core V	MTH-365	Optimization techniques	3	3	45	10	40
	Core VI	MTH-366	Dynamical systems	3	3	45	10	40
SEC	Skill Based	MTH-360	Vector analysis	2	2	30	10	40
DSC	Core (Practical)	MTH-367	Practical course based on MTH-361 and MTH-362	2	4 / batch	60	10	40
		MTH-368	Practical course based on MTH-363 and MTH-364	2	4 / batch	60	10	40
		MTH-369	Practical course based on MTH-365 and MTH-366	2	4 / batch	60	10	40

DSC: Discipline Specific Core Courses/Core Practical; **SEC:** Skill Enhancement Course; **Int :** Internal examination; **Ext :** External examination

T.Y. B.Sc. (Mathematics): Semester-V
Discipline Specific Core (DSC) Course
MTH-351: Methods of real analysis

Total Hours: 45 **Credits: 3**

Course objectives:

- To study Sequences of real numbers, series of functions.
- To study of Fourier series, Theory of Uniform convergence of sequences of functions and Cauchy's criteria for uniform convergence of sequences of functions.

Course outcomes:

After successful completion of this course, a student is expected to:

- Decide, whether a given series is convergent or divergent.
- Use different tests for absolute convergence.
- Understand Fourier series for even and odd functions.
- Understand sine and cosine series in half range.

Unit-I: Sequences of real numbers and sequences of functions **(12 h)**

1.1 Definition of a sequence and a subsequence of real numbers.
1.2 Convergence and divergence of sequences of real numbers
1.3 Bounded sequences
1.4 Monotone sequences of real numbers
1.5 Point wise convergence of sequences of functions
1.6 Uniform convergence of sequences of functions
1.7 Cauchy's criteria for uniform convergence of a sequence of functions
1.8 Consequences of uniform convergence

Unit-II: Series of real numbers **(11 h)**

2.1 Convergence and divergence
2.2 Series with non-negative terms
2.3 Alternating Series
2.4 Conditional convergence and absolute convergence
2.5 Rearrangement of series
2.6 Tests for absolute convergence
2.7 Series whose terms form non-increasing sequence

Unit-III: Series of functions **(11 h)**

3.1 Pointwise convergence of a series of functions
3.2 Uniform convergence of a series of functions
3.3 Integration and differentiation of a series of functions
3.4 Abel's Summability

Unit-IV: Fourier series in the range $[-\pi, \pi]$ **(11 h)**

4.1 Fourier series and Fourier coefficients
4.2 Dirichlet's condition of convergence (statement only)
4.3 Fourier series for even and odd functions
4.4 Sine and Cosine Series in half range

Recommended Books:

- Goldberg R.R.(1976), *Methods of Real Analysis*, 2nd Edition, Oxford and IBH Publishing Co.Pvt. Ltd., (Unit 1:- 2.1, 2.3, 2.4, 2.5, 2.6, 9.1, 9.2, 9.3, Unit 2:- 3.1, 3.2, 3.3, 3.4, 3.5, 3.6, 3.7, Unit 3:- 9.4, 9.5, 9.6)
- Spigel M. R.(1965), *Laplace Transform and Fourier series*, Schaum series, Mc. Graw Hill,. (Unit 4)

Reference Book:

- Malik S. C. And Arora Savita (2010), *Mathematical Analysis*, 4th edition, New Age International Pvt. Ltd., New Delhi.

Method of Teaching: Classroom teaching, Lecture method, ICT based teaching

T.Y. B.Sc. (Mathematics): Semester-V
Discipline Specific Core (DSC) Course
MTH-352: Integral calculus

Total Hours: 45 **Credits: 3**

Course objectives:

- To study Riemann Integration and its properties.
- To study Mean value theorems of integral calculus.
- To study Improper integrals with finite limit and infinite limit.
- To study Beta and Gamma Integrals.

Course outcomes:

After successful completion of this course, a student is expected to:

- Understand the concept of Riemann Integration.
- Understand Improper integrals with finite limit and infinite limit their properties.
- Learn the concepts of Beta and Gamma Integrals.

Unit-I: Riemann integration **(11 h)**

1.1 Definition and Existence of the Integral, The meaning of $\int_a^b f\,dx$ when $a \leq b$, Inequalities for integrals
1.2 Refinement of partitions
1.3 Darboux's Theorem (without proof)
1.4 Conditions of integrability
1.5 Integrability of the sum and difference of integrable functions
1.6 The integral as a limit of sum (Riemann Sums) and the limit of sum as the integral and its applications
1.7 Properties of integrable functions

Unit-II: Mean value theorems of integral calculus **(11 h)**

2.1 The First mean value theorem
2.2 The generalized first mean value theorem
2.3 Abel's lemma (without proof)
2.4 Second mean value theorem. Bonnets form and Karl Weierstrass form

Unit-III: Improper integrals **(12 h)**

3.1 Integration of unbounded functions with finite limits of Integral
3.2 Comparison Test for convergence at a of $\int_a^b f\,dx$
3.3 Convergence of the improper integrals $\int_a^b \frac{dx}{(x-a)^n}$
3.4 Cauchy's general test for convergence at the point a of $\int_a^b f\,dx$
3.5 Absolute convergence of the improper integrals $\int_a^b f\,dx$
3.6 Convergence of the integral with infinite range of Integration
3.7 Comparison Test for convergence at ∞
3.8 Convergence at a of $\int_a^\infty \frac{dx}{x^n}\,(a > 0)$
3.9 Cauchy's General Test for convergence at ∞

3.10 Absolute convergence of $\int_a^\infty f\,dx$

3.11 Test for absolute convergence of $\int_a^\infty f\,dx$

3.12 Abel's Test and Dirichlet's Test for convergence of $\int_a^\infty f\,dx$

Unit-IV: Beta and Gamma integrals **(11 h)**

4.1 Convergence of Beta and Gamma Integrals

4.2 Properties of Beta and Gamma Functions

4.3 Relation between Beta and Gamma Functions

4.4 Duplication Formula

4.5 Evaluation of integrals using Beta and Gamma Integrals

Recommended Book:

- Malik S. C. And Arora Savita (2000), *Mathematical Analysis*, 2^{th} edition, New Age International Pvt. Ltd., New Delhi. Chapter 9: 1 to 13, Chapter 11: 1 to 5.

Reference Book:

- Goldberg R.R.(1976), *Methods of Real Analysis*, 2^{nd} Edition, Oxford and IBH Publishing Co.Pvt. Ltd.

Method of Teaching:

- Classroom teaching, Lecture method, ICT based teaching

T.Y. B.Sc. (Mathematics): Semester-V
Discipline Specific Core (DSC) Course
MTH-353: Modern algebra

Total Hours: 45 **Credits: 3**

Course objectives:

- To gain the knowledge of basic concepts related to a group such as subgroups, normal, isomorphism.
- To understand basic concepts related to a ring such as an ideal, isomorphism of rings and polynomial rings.

Course outcomes:

After successful completion of this course, a student is expected to:

- Know the use permutation groups.
- Know normal subgroups and group isomorphisms.
- Know ideals in rings, quotient rings and isomorphism of rings.
- Know polynomial rings and irreducibility of polynomials.

Unit-I: Permutation groups **(11 h)**

1.1 Definitions: Permutation, Cycle, Transposition
1.2 Permutations as a product of disjoint cycles and transpositions
1.3 Even and odd permutations
1.4 Permutation Groups, Alternating Groups

Unit-II: Normal subgroups and isomorphism of groups **(11 h)**

2.1 Normal Subgroup
2.2 Criterions for a subgroup to be a normal subgroup
2.3 Union and Intersection of normal subgroup
2.4 Quotient Group
2.5 Simple Group
2.6 Commutator subgroup
2.7 Revision of Homomorphism and Isomorphism of Groups
2.8 Isomorphism theorems for groups and examples
2.9 Cayley's theorem, Theorem: $o(A_n) = \frac{o(S_n)}{2}$

Unit-III: Ideals, quotient rings and isomorphism of rings **(12 h)**

3.1 Revision of Ring, integral domain, field and basic properties
3.2 Characteristics of a ring
3.3 Subrings, ideals, left ideals, right ideals, principal ideals, prime and maximal ideals.
3.4 Quotient rings
3.5 Quotient Field (Definition & Examples only)
3.6 Homomorphism and isomorphism of rings

Unit-IV: Polynomial rings **(11 h)**

4.1 Definition and Properties of polynomial rings
4.2 Roots of Polynomials
4.3 Factorization of Polynomials
4.4 Division Algorithm for Polynomials
4.5 Eisenstein's Criterion

Recommended Books:

- Gopalakrishnan N.S.(2003), *University Algebra* (2nd Revised Edition), New Age International Publishers,
  Chapter-1: Art.-1.7, 1.8, 1.9, 1.11; **Chapter-2**: Art.- 2.2, 2.3,2.4,2.5, 2.6, 2.7,2.8,2.9,2.14,2.15
- Fraleigh J.B.(2003), *A First Course in Abstract Algebra* (3rd Edition), Narosa Publishing House.
  Chapter-30: Art.-30.1, 30.2, 30.3; **Chapter-31:** Art.-31.1, 31.2.

Reference Books:

- Herstein I. N.(1964), *Topics in Algebra* (2nd Edition), Vikas Publishing House Pvt. Ltd. New Delhi.
- Khanna V. K. And Bhambri S. K.(2008), *A course in Abstract Algebra* (3rd Edition), Vikas Publishing House Pvt. Ltd. New Delhi.

Method of Teaching:

- Classroom teaching, Lecture method, ICT based teaching

T.Y. B.Sc. (Mathematics): Semester-V
Discipline Specific Core (DSC) Course
MTH-354: Lattice theory

Total Hours: 45 **Credits: 3**

Course objectives:

- To study the concepts of a poset and a lattice.
- To study the diagrammatic representation of lattice.
- To study the terms Maximal element, Minimal element, Greatest element, Least elements.
- To study the concept of ideals and its properties.
- To study homomorphism of lattices.
- To study modular and distributive lattice and their inter-relation.
- To study complemented and relatively complemented lattice.

Course outcomes:

After successful completion of this course, a student is expected to:

- Understand the structure of poset and lattice.
- Represent lattice in diagrammatic form.
- Understand the terms Maximal element, Minimal element, Greatest element, Least elements.
- Learn the concepts of ideals and their properties.
- Learn the concepts of homomorphism.
- Understand modular and distributive lattice and their interrelation.
- Understand complemented and relatively complemented lattice.

Unit-I: Posets **(12 h)**

1.1 Posets and Chains
1.2 Diagrammatical Representation of posets
1.3 Maximal and Minimal elements of subset of a poset, Zorn's Lemma (Statement only)
1.4 Supremum and infimum
1.5 Poset isomorphism
1.6 Duality Principle.

Unit-II: Lattices **(11 h)**

2.1 Two definitions of lattice and equivalence of two definitions
2.2 Modular and Distributive inequalities in a lattice
2.3 Sublattice and Semilattice
2.4 Complete lattice

Unit-III: Ideals and homomorphisms **(11 h)**

3.1 Ideals, Union and intersection of Ideals
3.2 Prime Ideals

Ftandh.

3.3 Principal Ideals
3.4 Dual Ideals
3.5 Principal dual Ideals
3.6 Complements, Relative Complements
3.7 Homomorphisms, Join and meet homomorphism

Unit-IV: Modular and distributive lattices **(11 h)**

4.1 Modular lattice
4.2 Distributive lattice
4.3 Sublattice of Modular lattice
4.4 Homomorphic image of Modular lattice
4.5 Complemented and Relatively complemented lattice

Recommended Book:

- Khanna Vijay K. (2004), *Lattices and Boolean Algebra*, (Second Edition), Vikas Publ. Pvt. Ltd. **Chapter-2:** Art.-30.1, 30.2, 30.3; **Chapter-3:** Art.-31.1, 31.2; **Chapter-4:** Art.-31.1, 31.2.

Reference Book:

- Gratzer George (2013), *General Lattice Theory*, (Second Edition), Birkhauser.

Method of Teaching:

- Classroom teaching, Lecture method, ICT based teaching

T.Y. B.Sc. (Mathematics): Semester-V
Discipline Specific Core (DSC) Course
MTH-355: Number theory

Total Hours: 45 **Credits: 3**

Course objectives:

- To know scope and importance of elementary number theory and applications.
- To study problems in many branches of Mathematics such as Algebra, Field Theory, Algebraic number theory, Cryptography etc.
- To know properties of prime numbers, congruences.

Course outcomes:

After successful completion of this course, a student is expected to:

- Understand prime numbers, theory of Congruences and their properties which is one of the building blocks of pure mathematics.
- Explain Diophantine equations, Euler and Fermat theorem.
- Explain concepts of perfect numbers, Mersenne numbers and Fermat's numbers, Fibonacci numbers.
- Learn basic properties of Finite continued fractions.

Unit-I: Prime numbers and Diophantine Equation **(11 h)**

1.1 The Fundamental Theorem of Arithmetic
1.2 The Sieve of Eratosthenes
1.3 The Goldbach Conjecture
1.4 The Diophantine Equation $ax + by = c$

Unit-II: The theory of Congruences **(12 h)**

2.1 Basic Properties of Congruence
2.2 Binary and decimal representations of integers
2.3 Special Divisibility Tests
2.4 Linear Congruences and the Chinese Remainder Theorem
2.5 Fermat's Factorization Method
2.6 The Fermat's Little Theorem and pseudoprimes
2.7 Wilson's Theorem

Unit-III: Perfect numbers **(11 h)**

3.1 Perfect Numbers
3.2 Mersenne Numbers
3.3 Fermat's Numbers

Unit-IV: Fibonacci numbers and Finite continued fractions **(11 h)**

4.1 The Fibonacci Sequence
4.2 Certain Identities Involving Fibonacci Numbers
4.3 Finite continued fractions

Recommended Book:

- Burton David M.(2007), *Elementary Number Theory*, (Sixth Edition), Tata McGraw-Hill Edition, New Delhi.
  Ch.3 : 3.1 to 3.3, Ch . 2 : 2.5, Ch.4 : 4.2 to 4.4, Ch.5 : 5.2 to 5.4, Ch.11 : 11.2 to 11.4, Ch 14 : 14.2 to 14.3, Ch 15: 15.2

Reference Books:

- Apostol T. M.(1972), *Introduction to Analytic Number Theory*,(student Edition), Springer International,.
- Hari Kishan (2019), *Number Theory*, Krishna Prakashan Media (p) Ltd, Meerat.

Method of Teaching:

- Classroom teaching, Lecture method, ICT based teaching

T.Y. B.Sc. (Mathematics): Semester-V
Discipline Specific Core (DSC) Course
MTH-356: Fourier transforms

Total Hours: 45 **Credits: 3**

Course objectives:

- To know the concept of Fourier transform, Inverse Fourier transform, Z-transform and inverse Z-transform.
- To study Fourier sine transform, Fourier cosine transform, Finite Fourier sine transform and Finite Fourier cosine transforms.
- To know the applications of Fourier transform and Z-transform.

Course outcomes:

After successful completion of this course, a student is expected to:

- Apply transform techniques to signals and systems.
- Analyze Linear Time-Invariant systems by transform techniques.
- Understand different solution techniques.

Unit-I: Fourier Transforms **(12 h)**

1.1 Fourier integral formula and equivalent form of Fourier integral formula,
1.2 Fourier cosine and sine integrals
1.3 Fourier transforms and inverse Fourier transforms
1.4 Fourier transforms of generalized functions
1.5 Properties of Fourier transforms
1.6 Addition Theorem and Modulation Theorem

Unit-II: Fourier Cosine and Sine Transforms **(11 h)**

2.1 Riemann-Lebesgue Lemma, Derivative Theorem and Convolution Theorem
2.2 General Parseval's relation
2.3 Fourier cosine and sine transforms
2.4 Inverse cosine and sine transforms
2.5 Properties of Fourier cosine and sine transforms.

Unit-III: Application of Fourier Transforms **(10 h)**

3.1 Finite Fourier sine and cosine transforms
3.2 Properties of finite Fourier sine and cosine Transforms
3.3 Solutions of integral equations
3.4 Application of Fourier transforms to boundary value problem

Unit-IV: Z -Transforms **(12 h)**

4.1 Definition of Z-transforms
4.2 Properties of Z-transforms
4.3 Convolution Theorem, Parseval's Formula, Initial Value Theorem and Final Value Theorem
4.4 Z-transform of partial derivatives
4.5 Inverse Z-transform
4.6 Applications of Z-transforms to finite difference equations

Tundhi

4.7 Summation of Infinite Series

Recommended Books:

- Debnath Lokenath, Bhatta Dambaru (2007), *Integral Transforms and Their Applications* (Second Edition), Taylor and Francis Group, LLC.
  (Chapter 2: 2.1-2.5, 2.11-2.16, Chapter 10: 10.1-10.4)
- Bracewell Ronald N. (2000), *The Fourier Transform and Its Applications* (Third Edition), McGraw-Hill. (Chapter 6)

Reference Book:

- Davies Brian (2002) , *Integral Transforms and Their Applications* (Third Edition), Springer-Verlag, New York.

Method of Teaching:

- Classroom teaching, Lecture method, ICT based teaching

T.Y. B.Sc. (Mathematics): Semester-V
Skill Enhancement Course (SEC)
MTH-350: Laplace transforms

Total Hours: 30 **Credits: 2**

Course objectives:

- To know scope and importance of Laplace transforms and its applications to differential equations.
- To explain the concept of Laplace transforms and Inverse Laplace transforms with their properties.
- To discuss Convolution theorem and evaluation of inverse Laplace transform by using convolution theorem.

Course outcomes:

After successful completion of this course, a student is expected to:

- Understand basic concepts on Laplace and Inverse Laplace transforms.
- Understand convolution theorem.
- Understand use of Laplace transform in solving Differential Equations.

Unit-I: Laplace Transforms **(8 h)**

1.1 Definition and existence of Laplace transform.
1.2 Laplace transforms of elementary functions and validity.
1.3 Properties of Laplace transform.
1.4 Laplace transforms of derivatives.
1.5 Laplace transforms of real integrals.
1.6 Multiplication by t^n.
1.7 Division by t.
1.8 Definition and simple properties of Beta and Gamma functions (without proof).

Unit-II: Inverse Laplace Transform **(8 h)**

2.1 Definition and use of table.
2.2 Properties of inverse Laplace Transforms
2.3 Linearity
2.4 Inverse Laplace transforms of derivatives.
2.5 Inverse Laplace transforms of integrals.
2.6 Multiplications by s.
2.7 Division by s.

Unit-III: Convolution Theorem **(7 h)**

3.1 Laplace transforms of periodic functions.
3.2 Convolution theorem (without proof).
3.3 Evaluation of inverse Laplace transform by convolution theorem.
3.4 Use of partial fractions.

Unit-IV: Applications to Differential Equations **(7 h)**

4.1 Solution of linear differential equation with constant coefficients by using
4.2 Laplace transforms.

4.3 Laplace transforms of Heaviside Unit step functions.
4.4 Laplace transforms of Dirac-Delta functions.

Recommended Books:

- Spiegel Murry R. (1965), *Theory and problems of Laplace transforms*, Schaum's Outline Series, McGraw Hill Ltd, New York.
- Vasishtha A. R., Gupta R. K. (2014), *Integral transforms*, Krishna Prakashan Media (P) Ltd (Meerut).

Method of Teaching:

- Classroom teaching, Lecture method, ICT based teaching

T.Y. B.Sc. (Mathematics): Semester-V
Discipline Specific Core (DSC) Course
MTH-357: Practical course based on MTH-351 and MTH-352

Total Hours: 60 **Credits: 2**

Course objectives:

- To develop analytical and computational skills.
- To know problem solving skills in Theory of sequences and series.
- To get hands on training for solving problems of Riemann integrals.
- To know the concept of mean value theorem
- To get understanding about improper integrals, beta function and gamma function.

Course outcomes:

After successful completion of this course, a student is expected to:

- Explain concepts and solve problems on sequence, series and Fourier series.
- Solve problems related to Riemann integration.
- Solve problems on improper integrals
- Solve problems on beta functions and gamma functions
- Solve problems on mean value theorem.

Sr. No.	Topic Particular	Hours
1	Sequences of real numbers	4
2	Sequences of functions	4
3	Series of real numbers	4
4	Rearrangements and absolute convergence	4
5	Pointwise and uniform convergence of series of functions	4
6	Integration, differentiation of series of functions and Abel's summability	4
7	Fourier series in the range $[-\pi, \pi]$	4
8	Half range sine and cosine series	4
9	Riemann Integration-I	4
10	Riemann Integration-II	4
11	Mean Value Theorem-I	4
12	Mean Value Theorem-II	4
13	Improper integral for finite limit	4
14	Improper integral for infinite limit	4
15	Beta and Gamma Integrals	4

Practical No. 1: Sequence of real numbers

1. Give an example of a sequence $\{s_n\}_{n=1}^{\infty}$ of real numbers for which $\{|s_n|\}_{n=1}^{\infty}$ converges but $\{s_n\}_{n=1}^{\infty}$ does not.

2. Discuss the convergence of the sequence $\left\{\sqrt{n+1} - \sqrt{n}\right\}_{n=1}^{\infty}$.
3. Prove that every convergent sequence is bounded.
4. Show by an example that every bounded sequence need not be convergent.
5. Discuss the convergence of the sequence $\left\{\left(1+\frac{1}{n}\right)^{n}\right\}_{n=1}^{\infty}$.

Practical No. 2: Sequence of functions

1. Test the uniform convergence for the sequence of functions, $f_n(x) = x^n$ for all $x \in [0,1]$, $n \in \mathbb{N}$.
2. Test the uniform convergence for the sequence of functions, $g_n(x) = \frac{x}{1+nx}$ for all $0 \leq x < \infty, n \in \mathbb{N}$.
3. Let $f_n(x) = \frac{x^n}{1+x^n}$ for all $x \in [0,1], n \in \mathbb{N}$. Show that $\{f_n\}_{n=1}^{\infty}$ converges pointwise on $[0,1]$. If $\lim_{n\to\infty} f_n(x) = f(x)$, then does there exist a positive integer N such that $|f_n(x) - f(x)| < \frac{1}{4}\ \forall\, n \geq N, \forall\, x \in [0,1]$ simultaneously.
4. Show that the sequence of functions $f_n(x) = \frac{nx}{1+n^2x^2}$ for all $x \in \mathbb{R}$, $n \in \mathbb{N}$, does not converge uniformly on $\mathbb{R}$ although it converges pointwise to 0.
5. Let $f_n(x) = \frac{\sin nx}{n}$ for all $x \in [0,1], n \in \mathbb{N}$. Show that $\{f_n\}_{n=1}^{\infty}$ converges uniformly to 0 on $[0,1]$ but $\{f'_n\}_{n=1}^{\infty}$ does not converge even pointwise to 0 on $[0,1]$.

Practical No. 3: Series of real numbers

1. Discuss the convergence of the series $\sum_{n=1}^{\infty} n$.
2. Test the convergence for the series: a) $\sum_{n=1}^{\infty} \frac{n+1}{n+2}$ b) $\sum_{n=1}^{\infty} \frac{n+1}{10^{10}(n+2)}$.
3. Show that the series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}$ is convergent.
4. Test the convergence of the series $(1-2) - (1-2^{1/2}) + (1-2^{1/3}) - (1-2^{1/4}) + \ldots$
5. Discuss the convergence of the series $1 + 2x + 3x^2 + 4x^3 + \cdots$.

Practical No. 4: Rearrangements and absolute convergence

1. If $\sum_{n=1}^{\infty} a_n$ is a series of non-negative numbers which converges to $A \in \mathbb{R}$ and $\sum_{n=1}^{\infty} b_n$ is any rearrangement of $\sum_{n=1}^{\infty} a_n$, then show that $\sum_{n=1}^{\infty} b_n$ converges to A.
2. Discuss the convergence of the series $\sum_{n=1}^{\infty} \frac{n^4}{n!}$.
3. Test the convergence of the series $\sum_{n=1}^{\infty} \frac{\left(1+\frac{1}{n}\right)^{2n}}{e^n}$.
4. For what values of x, does $\sum_{n=1}^{\infty} \frac{1}{n^x}$?
5. Prove that for any real number x the series $\sum_{n=3}^{\infty} \frac{1}{(\log n)^x}$ diverges.

Practical No. 5: Pointwise and uniform convergence of series of functions

1. Show that the series $\sum_{n=1}^{\infty} \frac{\sin nx}{n^2}$ converges uniformly on $\mathbb{R}$.
2. Show that the series $\sum_{n=1}^{\infty} \frac{1}{n^2+x^2}$ converges uniformly on $[0,\infty)$.
3. Show that the series $\sum_{n=1}^{\infty} \frac{nx^2}{n^3+x^3}$ is uniformly convergent on $[0,A]$ for any $A > 0$.
4. Test the convergence of the series $\sum_{n=1}^{\infty} e^{-nx} x^n \quad 0 \leq x \leq 10$.

5. If $\sum_{n=0}^{\infty} a_n < \infty$, then prove that the series $\sum_{n=0}^{\infty} a_n x^n$ converges uniformly for $[0,1]$.

Practical No. 6:
Integration, differentiation of series of functions and Abel's summability

1. Without finding sun of $f(x)$ of series, $1+\frac{x^2}{1!}+\frac{x^4}{2!}+\frac{x^6}{3!}+\cdots+\frac{x^{2n}}{n!}+\cdots\ (-\infty<x<\infty)$, show that $f'(x)=2xf(x)$.
2. From the series $sinx = x-\frac{x^3}{3!}+\frac{x^5}{5!}+\cdots$, deduce that $cosx = 1-\frac{x^2}{2!}+\frac{x^4}{4!}-\cdots$.
3. Show that the series whose $S_n(x)=\frac{1}{nx+1}$ can be integrated term by term in $[0,1]$ although it is not uniformly convergent in $[0,1]$.
4. Show that the series $S_n(x)=nx(1-x)^n$ can be integrated term by term in $[0,1]$.
5. Show that the series $1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\cdots$ is Abel summable.

Practical No. 7: Fourier series in the range $[-\pi,\pi]$

1. If f is a bounded and integrable function on $[-\pi,\pi]$ and a_n, b_n are its Fourier coefficients, then prove that $\sum_{n=1}^{\infty}({a_n}^2+{b_n}^2)$ converges.
2. Find Fourier series in $[-\pi,\pi]$ for $f(x)=\begin{cases}0 & if -\pi<x<0\\ x & if \quad 0<x<\pi\end{cases}$.
3. Find Fourier series for $f(x)=x+x^2$ in $[-\pi,\pi]$ and hence deduce that $\frac{\pi^2}{6}=\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\cdots$
4. Expand $f(x)=|x|$ in a Fourier series $[-\pi,\pi]$ and hence deduce that $\frac{\pi^2}{8}=\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\cdots$.
5. Find Fourier series for $f(x)=x^2$ in $[-\pi,\pi]$.

Practical No. 8: Half range sine and cosine series

1. Find half range sine series for $f(x)=x$ in $[0,\pi]$.
2. Find half range sine series for $f(x)=cos2x$ in $[0,\pi]$.
3. Find half range cosine series for $f(x)=x$ in $[0,\pi]$.
4. Find half range cosine series for $f(x)=x^2$ in $[0,\pi]$.
5. Find half range cosine series for $f(x)=sin3x$ in $[0,\pi]$.

Practical No. 9: Riemann Integration-I

1. Let $f(x)=x^2$ defined on $[0,k]$. Find
   a) $U(P,f)$ b) $L(P,f)$ and show that $\int_0^k f(x)\,dx=\frac{k^3}{3}$.
2. Find the upper and lower integral for the function defined on [0,1]as
   $$f(x)=\begin{cases}\sqrt{1-x^2} & ,\text{ when } x \text{ is rational}\\ 1-x & ,\text{when } x \text{ is irrational}\end{cases}.$$
3. The function $f(x)$ defined on $\left[0,\frac{\pi}{4}\right]$ as $f(x)=\begin{cases}\cos x & ,\text{when } x \text{ is rational}\\ \sin x & ,\text{when } x \text{ is irrational}\end{cases}$.
   Show that $f(x)\notin R\left[0,\frac{\pi}{4}\right]$.
4. Show that the function defined as $f(x)=\begin{cases}\frac{1}{2^n} & if\ \frac{1}{2^{n+1}}<x\le\frac{1}{2^n} \quad ,n=0,1,2,\cdots\\ 0 & if\ x=0\end{cases}$

is integrable on $[0,1]$ and evaluate $\int_0^1 f(x)\, dx$.

5. A function defined on $[0,1]$ as $f(x) = \frac{1}{a^{r-1}}$, if $\frac{1}{a^r} < x \le \frac{1}{a^{r-1}}$ where a is an integer greater than 2, and, $r = 1,2,3,\cdots$ Show that
   a) $\int_0^1 f(x)\, dx$ exists. b) $\int_0^1 f(x)\, dx = \frac{a}{a+1}$.

Practical No. 10: Riemann Integration-II

1. Evaluate $lim_{n\to\infty} \frac{1}{n}\left[\frac{1^{235}+2^{235}+3^{235}+\cdots+n^{235}}{n^{235}}\right]$.
2. Evaluate $lim_{n\to\infty} \left[\frac{n^n}{n!}\right]^{\frac{1}{n}}$.
3. Using Riemann integral as a limit of sum evaluate $\int_1^2 (5x+7)dx$.
4. Using Riemann sum evaluate $\int_{-1}^1 |x| dx$.
5. Give an example of a function f such that $|f|$ is integrable on $[a,b]$ but f is not integrable on $[a,b]$.

Practical No. 11: Mean Value Theorem-I

1. Using Mean Value Theorem. Prove that $\frac{\pi^3}{24} \le \int_0^\pi \frac{x^2}{5+3\cos x} dx \le \frac{\pi^3}{3}$
2. Show that $\frac{1}{2} \le \int_0^1 \frac{dx}{\sqrt{4-x^2+x^3}} dx \le \frac{\pi}{6}$
3. If $a > 0$, Show that $ae^{-a^2} < \int_0^{-a^2} e^{-x^2}\, dx < \tan^{-1} a$
4. Show that $\frac{1}{\sqrt{2}} < \int_0^{\frac{1}{\sqrt{2}}} \frac{dx}{\sqrt{1-x^4}} < \frac{\pi}{4}$ for all $x \in \left[0, \frac{1}{\sqrt{2}}\right]$
5. Show that $lim_{n\to\infty} \int_0^1 \frac{nf(x)}{1+n^2x^2} dx = \frac{\pi}{2} f(0)$.

Practical No. 12: Mean Value Theorem-II

1. Verify second Mean Value Theorem for the function $f(x) = x$ and $g(x) = e^x$
2. Verify second Mean Value Theorem for the function $f(x) = x$and $g(x) = \sin x$.
3. Show that there exist $\xi_1, \xi_2 \in [0,1]$ such that $\int_0^1 \frac{\sin \pi x}{x^2+1} dx = \frac{2}{\pi(\xi_1^2+1)} = \frac{\pi}{4}(\sin \pi\xi_2)$.
4. If $0 < a < b$ then show that a) $\left|\int_a^b \frac{\sin x}{x} dx\right| \le \frac{2}{a}$ b) $\left|\int_a^b \sin(x^2) dx\right| \le \frac{1}{a}$.
5. Prove by an example that second mean value theorem not hold for non-monotonic function f.

Practical No. 13: Improper integral for finite limit

1. Show that $\int_0^2 \frac{\log x}{\sqrt{2-x}} dx$ is convergent.
2. Discuss the convergence of $\int_1^2 \frac{\sqrt{x}}{\log x} dx$.
3. Test the convergence of $\int_0^1 \frac{dx}{x^{1/2}(1-x)^{1/2}}$.
4. Show that the integral $\int_0^{\pi/2} \log \sin x\, dx$ is convergent and hence evaluate it.
5. Show that $\int_0^1 x^{m-1}(1-x)^{n-1}\, dx$ exists if and only if $m, n \ge 0$.

Practical No. 14: Improper integral for infinite limit

1. Examine the convergence of $\int_0^\infty \frac{x^2}{\sqrt{x^5+1}} dx$.

2. Show that $\int_0^{\infty} \frac{\sin^2 x}{x^2} dx$ is convergent.
3. Test the convergence of the integral $\int_0^{\infty} \frac{x \tan^{-1} x}{(1+x^4)^{1/3}} dx$.
4. Show that the integral $\int_0^{\infty} x^{m-1}e^{-x}\, dx$ is convergent if and only if $m > 0$.
5. Using Cauchy s Test, show that $\int_0^{\infty} \frac{\sin x}{x} dx$ is convergent.

Practical No. 15: Beta and Gamma Integrals

1. Show that $\Gamma(n) = \int_0^{\infty} \left(\log \frac{1}{y}\right)^{n-1} dy$.
2. Show that $\int_0^{\infty} e^{-x^2} dx = \frac{\sqrt{\pi}}{2}$
3. Prove that $\int_0^{\infty} \frac{x^{m-1}}{(1+x)^{m+n}} dx = \beta(m,n)$.
4. Prove that $\left\{\int_0^1 \frac{x^2 dx}{\sqrt{1-x^4}}\right\}\left\{\int_0^1 \frac{dx}{\sqrt{1-x^4}}\right\} = \frac{\pi}{4}$
5. Prove that $\int_0^{\infty} xe^{-x^8} dx \int_0^{\infty} x^2 e^{-x^4} dx = \frac{\pi}{16\sqrt{2}}$.

Recommended Books:

- Goldberg R.R.(1976), *Methods of Real Analysis*, 2nd Edition, Oxford and IBH Publishing Co.Pvt. Ltd.
- Spigel M. R.(1965), *Laplace Transform and Fourier series*, Schaum series, Mc. Graw Hill.

Reference Book:

- Malik S. C. And Arora Savita (2010), *Mathematical Analysis*, 4th edition, New Age International Pvt. Ltd., New Delhi.

Method of Teaching:

- Classroom teaching, Lecture method, ICT based teaching

T.Y. B.Sc. (Mathematics): Semester-V
Discipline Specific Core (DSC) Course
MTH-358: Practical course based on MTH-353 and MTH-354

Total Hours: 60 **Credits: 2**

Course objectives:

- To develop analytical and computational skills
- To get hands on training in solving problems of groups, rings and lattice theory.

Course outcomes:

After successful completion of this course, a student is expected to:

- Understand and solve problems on permutations, normal subgroups, ideals in rings, reducible and irreducible polynomial rings.
- Apply isomorphism theorems to solve problems on isomorphic groups and rings.
- Explain concepts and solve problems on posets, lattices, sublattices and dual ideals in lattices.
- Understand distributive and modular lattices and solve the problem onit.

Sr. No.	Topic Particular	Hours
1	Permutations and order of permutations	4
2	Permutations	4
3	Normal subgroups	4
4	Isomorphism theorems for groups	4
5	Characteristics of ring, ideals and subrings	4
6	Quotient rings, quotient fields and isomorphism of rings	4
7	Polynomial rings	4
8	Reducible and irreducible polynomials	4
9	Posets and chains	4
10	Diagrammatic representation of posets	4
11	Lattices	4
12	Sublattices, semilattices and complete lattices	4
13	Ideals and dual ideals	4
14	Complements, relative complements and homomorphisms	4
15	Modular and Distributive lattices	4

Practical No. 1: Permutations and order of permutations

1. Prepare a multiplication table of the permutations on set $A = \{1, 2, 3\}$ and show that S_3 is a group under the operation of permutation multiplication.
2. Find all even permutations in the permutation group S_4.

3. If $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 5 & 4 & 1 & 6 & 3 & 2 \end{pmatrix}$ and $\mu = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 3 & 4 & 1 & 6 & 5 \end{pmatrix}$ in S_6, then find
   i) σ^2 ii) μ^2 iii) $\sigma\mu$ iv) $\mu\sigma$ v) σ^{-1} vi) μ^{-1}.
4. If $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 6 & 4 & 7 & 5 & 2 & 3 & 1 \end{pmatrix}$ in S_7, then express σ as a product of transpositions. Is it an even permutation? Also find order of σ.
5. If $\mu = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 4 & 6 & 2 & 5 & 8 & 9 & 3 & 1 & 7 \end{pmatrix}$ in S_9, then find order of the permutation μ^{-1}.

Practical No. 2: Permutations

1. Find $\sigma^{-1}\rho\sigma$ where $\rho = (1\,3\,4)(5\,6)(2\,7\,8\,9)$ and $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 7 & 8 & 9 & 6 & 4 & 5 & 2 & 3 & 1 \end{pmatrix}$.
2. Find $\rho \in S_6$ such that $\rho\sigma\rho^{-1} = \mu$ where $\sigma = (1\,3)(4\,6\,2\,5)$, $\mu = (2\,4\,5\,1)(3\,6)$.
3. Find $\rho \in S_9$ such that $\rho\sigma\rho^{-1} = \mu$ where $\sigma = (2\,3)(1\,4\,6\,7\,8)(5\,9)$, $\mu = (1\,2\,4\,5\,6)(3\,7)(8\,9)$.
4. Show that there does not exists a permutation $\sigma \in S_8$ such that $\sigma(1\,5\,7)\sigma^{-1} = (1\,5)(2\,4\,6)$.
5. Show that there does not exists a permutation $\sigma \in S_8$ such that $\sigma(3\,6)\sigma^{-1} = (2\,5\,8)$.

Practical No. 3: Normal subgroups

1. Show by an example that union of two normal subgroups of a group G need not be a normal subgroup.
2. Find all normal subgroups of the group of quaternion's $Q = \{\pm 1, \pm i, \pm j, \pm k\}$.
3. Give an example of subgroups H, K of G such that H is normal in K and K normal in G but$H$ is not normal in G.
4. Let $G = GL(2, \mathbb{R}) = \{A : A$ is non-singular 2×2 matrix over $\mathbb{R}\}$, a group under usual matrixmultiplication and $H = SL(2, \mathbb{R}) = \{A \in G : |A| = 1\}$ a subgroup of G. Show that H is normal in G.
5. If $G = (\mathbb{Z}, +)$ and $N = (3\mathbb{Z}, +)$, then find the quotient group $\frac{G}{H}$.

Practical No. 4: Isomorphism theorems for groups

1. Let $\mathbb{R}^*$ be the multiplicative group of non-zero reals. Show that $\frac{GL(2,\mathbb{R})}{SL(2,\mathbb{R})} \cong \mathbb{R}^*$.
2. Let $G = \{1, -1\}$ be the group under multiplication. Show that the function $f: S_n \to G$ defined by $f(\sigma) = \begin{cases} 1 & \text{if } \sigma \text{ is even} \\ -1 & \text{if } \sigma \text{ is odd} \end{cases}$, is an onto group homomorphism. Find its kernel.
3. Show that $\mathbb{Z}_9$ is not a homomorphic image of $\mathbb{Z}_{16}$.
4. Show that the group $(\mathbb{Q}, +)$ is not isomorphic to $(\mathbb{Q}^+, \cdot)$.
5. Prove that there are only two (upto isomorphisms) groups of order six.

Practical No. 5: Characteristics of ring, ideals and subrings

1. Show that characteristics of a Boolean ring is two.
2. Find the characteristics for the rings i) $(\mathbb{Z}_n, +_n, \times_n)$ ii) $(\mathbb{Z}, +, \cdot)$.
3. Let R be a ring and $Z(R) = \{x \in R : xy = yx \ \forall y \in R\}$. Show that
   a) $Z(R)$ is a subring of R.
   b) If R is a division ring, then $Z(R)$ is a field.
4. Give an example of a right ideal in a ring which is not a left ideal.
5. Find all prime and maximal ideals in the ring $(\mathbb{Z}_{12}, +_{12}, \times_{12})$.

Gandhi

Practical No. 6: Quotient rings, quotient fields and isomorphism of rings

1. Show that intersection of two prime ideals in a ring need not be a prime ideal.
2. Find all prime ideals and maximal ideals in the ring $(\mathbb{Z}_{12}, +_{12}, \times_{12})$.
3. Let $R = C[0,1] = \{f : f$ is a continuous real valued function defined on $[0,1]\}$ be a ring under the operations $(f+g)(x) = f(x) + g(x)$ and $(f \cdot g)(x) = f(x)g(x)$ and $(\mathbb{R}, +, \cdot)$ be the ring of real under usual addition and multiplication. Show that $\left\{f \in R : f\left(\frac{1}{2}\right) = 0\right\}$ is a maximal ideal of R.
4. Show that $\mathbb{Z}_n \cong \frac{\mathbb{Z}}{n\mathbb{Z}}$ where $\mathbb{Z}$ is the ring of integers.
5. Find the field of quotients of $\mathbb{Z}[i]$.

Practical No. 7: Polynomial rings

1. Let $f(x) = 2x^3 + 4x^2 + 3x + 2$ and $g(x) = 3x^4 + 2x + 4$ in $\mathbb{Z}_5[x]$. Find
   a) $f(x) + g(x)$ b) $f(x) \cdot g(x)$ c) $\deg(f(x) \cdot g(x))$.
2. Let $f(x) = x^6 + 3x^5 + 4x^2 - 3x + 2$ and $g(x) = x^2 + 2x - 3$ be polynomials in $\mathbb{Z}_7[x]$. Find $q(x), r(x) \in \mathbb{Z}_7[x]$ such that $f(x) = g(x).q(x) + r(x)$ with $\deg(r(x)) < 2$.
3. a) Find all zeros of $f(x) = x^5 + 3x^3 + x^2 + 2x$ in $\mathbb{Z}_5$.
   b) Examine whether the polynomial $x^3 + 3x^2 + x - 4$ is irreducible over the field $(\mathbb{Z}_7, +_7, \times_7)$.
4. Express the polynomial $x^4 + 4$ as a product of linear factors in $\mathbb{Z}_5[x]$.
5. Give an example of polynomials $f(x)$ and $g(x)$ in a ring $\mathbb{Z}_6[x]$ such that
   $$\deg(f(x) \cdot g(x)) < \deg(f(x)) + \deg(g(x)).$$

Practical No. 8: Reducible and irreducible polynomials

1. Using Eisenstein's criteria show that the following polynomials are irreducible over the field of rationals. a) $x^2 + 8x - 2$ b) $8x^3 + 6x^2 - 9x + 24$.
2. Prove that the polynomial $1 + x + x^2 + \cdots + x^{p-1}$ is irreducible over the field of rationals, where p is a prime number.
3. Show that $\frac{\mathbb{Z}_5[x]}{\langle x^3+3x+2 \rangle}$ is a field.
4. Show that $\frac{\mathbb{Z}_3[x]}{\langle x^3+x+1 \rangle}$ is not an integral domain.
5. Show that $\langle x^2 + 1 \rangle$ is not a prime ideal of $\mathbb{Z}_2[x]$.

Practical No. 9: Posets and chains

1. Prove that intersection of two partial ordered relations on a non-empty set P is again a partial ordered relation on P.
2. Show that set of natural numbers $\mathbb{N}$ under 'usual less than or equal to relation' forms a poset.
3. Show that in a poset P,
   (i) $a < a$ for no $a \in P$ (ii) $a < b, b < c, where\ a, b, c \in P \Rightarrow a < c$.
4. Let A, B be posets. Denote $A \times B = \{(a, b) : a \in A, b \in B\}$. For $(a_1, b_1), (a_2, b_2) \in A \times B$, define $(a_1, b_1) \leq (a_2, b_2) \Leftrightarrow a_1 \leq a_2$ in A and $b_1 \leq b_2$ in B. Show that $(A \times B, \leq)$ is a poset.

5. List all chains containing two elements in a poset $\{1, 2, 3, 6\}$ under divisibility relation.

Practical No. 10: Diagrammatic representation of posets

1. Draw the Hasse diagram of D_{30}.
2. Find all maximal elements and minimal elements in the poset $\{2, 3, 4, 6\}$ under divisibility relation.
3. Let P be the poset $\{2, 3, 4, 6\}$ under divisibility relation. Show that P is self dual.
4. Prove that the two chains S and T under usual less than or equal to relation are dually isomorphic where $S = \left\{0, \dots, \frac{1}{n}, \dots, \frac{1}{3}, \frac{1}{2}, 1\right\}$, $T = \left\{0, \frac{1}{2}, \frac{2}{3}, \dots, \frac{n}{n+1}, \dots, 1\right\}$.
5. Let P, Q be posets. Prove that a function $f: P \to Q$ is an isomorphism if and only if f is isotone and has an isotone inverse.

Practical No. 11: Lattices

1. Show by an example that product of two chains need not be a chain.
2. Draw the lattice diagram of D_{20} and show that it is same as the product of two chains with three and two elements.
3. Let $(L, \leq)$ be a lattice and $a, b, c, d \in L$.If $\leq b, c \leq d$, then prove that $a \wedge c \leq b \wedge d$ and $a \vee c \leq b \vee d$.
4. Let $(L, \leq)$ be a lattice and $a, b, c \in L$. Prove that $a \wedge (b \vee c) \geq (a \wedge b) \vee (a \wedge c)$.
5. Let $(L, \leq)$ be a lattice and $a, b, c \in L$.If $a \geq b$, then prove that $a \wedge (b \vee c) \geq b \vee (a \wedge c)$.

Practical No. 12: Sublattices, semilattices and complete lattices

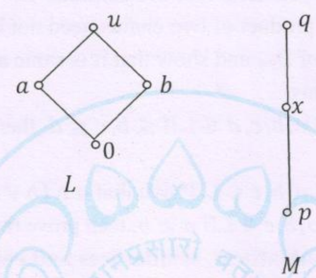
1. Show by an example that intersection of two sublattices of a lattice L may not be a sublattice of L.
2. Prove that a lattice L is a chain if and only if every non-empty subset of L is a sublattice.
3. Give an example of a meet semilattice which is not a join semilattice.
4. Show that the poset$\{2, 3, 6\}$ under divisibility relation is a join semilattice but not a meet semilattice.
5. Which of the following lattices is not a complete lattice?
   i) $\{1, 2, 3, \dots, 10\}$under usual less than or equal to relation.
   ii) $\mathbb{Z}$ under usual less than or equal to relation.

Practical No. 13: Ideals and dual ideals

1. Give an example of a sublattice of a lattice D_{10} which is not an ideal of D_{10}.
2. Show by an example that intersection of two ideals of a lattice L is an ideal of L.
3. Find all prime ideals in a lattice D_{18}.
4. Show by an example that union of two dual ideals of a lattice L need not be a dual ideal of L.
5. Let I be a prime ideal of a lattice L. Prove that $L - I$ is a dual prime ideal of L.

Practical No. 14:
Complements, relative complements and homomorphisms

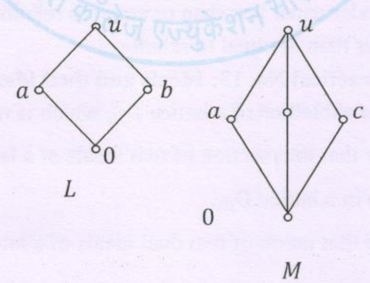
1. Show that the lattice D_{10} is a complemented lattice.
2. Prove that homomorphic image of a relatively complemented lattice is relatively complemented.
3. Let L, M be lattices and $0'$ be the least element in M. If $\theta: L \to M$ is an onto homomorphism, then prove that $Ker\theta$ is an ideal of L.
4. Let L, M be lattices and $\theta: L \to M$ be an onto homomorphism. If 0 is the least element in L, then, prove that $\theta(0)$ is the least element in M.
5. Let L, M be lattices as given below.



Let $\phi: L \to M$ be defined by $\phi(0) = p$, $\phi(a) = x$, $\phi(b) = x$, $\phi(u) = q$. Show that ϕ is neither a meet nor a join homomorphism.

Practical No. 15: Modular and Distributive lattices

1. Prove that Pentagonal lattice is not modular.
2. Show by an example that every modular lattice need not be distributive.
3. Show that the following lattices are modular.



4. If an element x has a complement in a distributive lattice $L$and $x \in [a, b] \subseteq L$, then show that x has a relative complement in $[a, b]$.
5. Prove that every complemented modular lattice is relatively complemented.

Recommended Books:

- Gopalakrishnan N.S.(2003), *University Algebra* (2^{nd} Revised Edition), New Age International Publishers.
- Fraleigh J.B.(2003), *A First Course in Abstract Algebra* (3^{rd} Edition), Narosa Publishing House
- Khanna Vijay K. (2004), *Lattices and Boolean Algebra*, (Second Edition), Vikas Publ. Pvt. Ltd.

Reference Books:

- Herstein I. N.(1964), *Topics in Algebra* (2^{nd} Edition), Vikas Publishing House Pvt. Ltd. New Delhi.
- Khanna V. K. And Bhambri S. K.(2008), *A course in Abstract Algebra* (3^{rd} Edition), Vikas Publishing House Pvt. Ltd. New Delhi.
- Gratzer George (2013), *General Lattice Theory*, (Second Edition), Birkhauser

Method of Teaching:

- Classroom teaching, Lecture method, ICT based teaching

T.Y. B.Sc. (Mathematics): Semester-V
Discipline Specific Core (DSC) Course
MTH-359: Practical course based on MTH-355 and MTH-356

Total Hours: 60 **Credits: 2**

Course objectives:

- To know problem solving skills in Theory of prime numbers and congruences.
- To know problem solving skills in Theory of Diophantine equations and finite continued fractions.
- To know problem solving skills in Theory of perfect, Mersenne, Fermat and Fibonacci numbers.
- To get hands on training in solving problems of Integral Transforms.

Course outcomes:

After successful completion of this course, a student is expected to:

- Understand and solve problems on prime numbers and congruences.
- Apply theorems of Euler and Fermat to solve problems.
- Explain concepts and solve problems on Fibonacci sequence.
- Apply concepts of finite continued fractions to solve problems on Diophantine equations.
- Solve the problems on Fourier transforms and Z-transforms
- Solve the boundary value problems by using Fourier transform
- Solve the finite difference equations by using Z-transform.

Sr. No.	Topic Particular	Hours
1	Prime Numbers	4
2	Goldbach Conjecture and Diophantine Equations	4
3	Congruences	4
4	Linear congruences	4
5	Perfect numbers	4
6	Mersenne and Fermat's Numbers	4
7	Fibonacci numbers	4
8	Finite continued fractions	4
9	Fourier Integrals	4
10	Fourier Transforms	4
11	Fourier sine and cosine transforms	4
12	Finite Fourier sine and cosine transforms	4
13	Applications of Fourier Transforms	4
14	Z-transforms	4
15	Inverse Z-transforms	4

Practical No. 1: Prime numbers

1. If $p \geq 5$ is a prime number, then show that $p^2 + 2$ is composite.
2. If $p \neq 5$ is an odd prime number, then prove that either $p^2 - 1$ or $p^2 + 1$ is divisible by 10.
3. Determine whether the integer 769 is prime by testing all primes $p \leq \sqrt{769}$ as possible divisors. Do the same for integer 1009.
4. Assuming that P_n is the n^{th} prime number, establish that the sum $\frac{1}{P_1} + \frac{1}{P_2} + \cdots + \frac{1}{P_n}$ is never an integer.
5. Obtain all prime numbers between 121 & 170 by Siev of Eratosthenes method.

Practical No. 2: Goldbach Conjecture and Diophantine Equations

1. Show that the sum of twin primes $P\ and\ p + 2$ is divisible by 12, $P > 3$
2. Find the smallest positive integer n such that $f(n) = n^2 + 21n + 1$ is a composite number.
3. Determine all solutions in the integers of the Diophantine equation $24x + 138y = 18$.
4. Determine all solutions in the positive integers of the Diophantine equation $18x + 5y = 48$.
5. Let n be the number of coins. If you make 77 strings of n, then 50 coins are short, but if you make 78 strings of n, it is exact. Find the smallest value of n.

Practical No. 3: Congruences

1. Find the remainder when $1! + 2! + 3! + 4! + \cdots + 99! + 100!$ is divided by 12.
2. What is the remainder when $1^5 + 2^5 + 3^5 + 4^5 + \cdots + 99^5 + 100^5$ is divided by 4?
3. Write the representation of 105 with base 3.
4. Find the last two digit number of the number 9^{9^9}.
5. Without performing the division, determine whether the integer 176521221 is divisible by 11.

Practical No. 4: Linear Congruences

1. Factorize 3901 by Fermat's factorization method.
2. Show that 341 is a pseudo prime number.
3. If $7 \nmid a$, then show that either $a^3 - 1\ or a^3 + 1$ is divisible by 7.
4. Find the remainder when 11^{104} is divided by 17.
5. Show that $18! \equiv -1\ (mod\ 437)$.

Practical No. 5: Perfect numbers

1. Show that the integer $n = 2^{10}(2^{11} - 1)$ is not perfect number.
2. If n is even perfect number, then prove that $\sum_{d|n} \frac{1}{d} = 2$.
3. Show that a perfect square can not be a perfect number.
4. Let n be an even perfect number > 6. Show that the sum of digits of n is congruent to $1\ modulo\ 9$.
5. Show that the product of two odd prime numbers is not a perfect number.

Practical No. 6: Mersenne and Fermat's Numbers

1. Prove that the Mersenne number M_{19} is prime. Hence show that $n = 2^{18}(2^{19} - 1)$ is perfect.
2. Find the Smallest Divisor of M_{11}.
3. Show that M_{29} is a composite number.
4. From the congruence $5.2^7 \equiv -1 \ (mod\ 641)$, deduce that $2^{32} + 1 \equiv 0 \ (mod\ 641)$. Hence $641 \mid F_5$.
5. For $n \geq 2$, show that the last digit of Fermat Number $F_n = 2^{2^n} + 1$ is 7.

Practical No. 7: Fibonacci numbers

1. Evaluate $\gcd(u_9, u_{12})$, $\gcd(u_{15}, u_{20})$, $\gcd(u_{24}, u_{36})$..
2. Find the Fibonacci numbers that divides both u_{24} *and* u_{36}.
3. For primes $P = 7,11,13,17$ verify that either u_{P-1}or u_{P+1} is divisible by p.
4. Prove that $u_{n+7} \equiv 8u_n (mod\ 13)$.
5. If $3|u_n$, then show that $9|\{(u_{n+1})^3 - (u_{n-1})^3\}$.

Practical No. 8: Finite continued fractions

1. Express $\frac{187}{57}$ as finite simple continued fraction.
2. Express $\frac{-19}{51}$ as finite simple continued fraction.
3. Determine the rational number represented by $[4 : 2, 1, 3, 1, 2, 4]$ as a finite simple continued fraction.
4. Represent $[-1: 2,1,6,1]$ as an odd number of partial denominators.
5. By using simple continued fraction, solve the Diophantine equation $19x + 51y = 1$.

Practical No. 9: Fourier Integrals

1. Find the Fourier integral representation of the function $f(x) = \begin{cases} 1, & |x| < 1, \\ 0, & |x| > 1. \end{cases}$
2. Find the Fourier integral representation of the function $f(x) = \begin{cases} 0, & x < 0, \\ e^{-x}, & x > 0, \\ \frac{1}{2}, & x = 0. \end{cases}$
3. By considering Fourier sine integral of e^{-mx} $(m > 0)$, prove that
$$\int_0^\infty \frac{k \sin kx}{m^2+k^2} dk = \begin{cases} -\frac{\pi}{2} e^{mx}, & x < 0, \\ \frac{\pi}{2} e^{mx}, & x > 0. \end{cases}$$
4. By considering Fourier cosine integral of e^{-mx} $(m > 0)$, prove that
$$\int_0^\infty \frac{\cos kx}{m^2+k^2} dk = \frac{\pi}{2m} e^{-mx}, m > 0, x > 0.$$
5. Find the Fourier cosine integral representation of the function
$$f(x) = \begin{cases} x^2, & 0 < x < a, \\ 0, & x > a. \end{cases}$$

Practical No. 10: Fourier Transforms

1. Find the Fourier transform of the function $f(x) = \left(1 - \frac{|x|}{a}\right) H\left(1 - \frac{|x|}{a}\right)$, where $H(x)$ is the Heaviside Unit step function defined by
$$H(x) = \begin{cases} 1, & x > 0, \\ 0, & x < 0. \end{cases} \quad \text{Or} \quad H(x-a) = \begin{cases} 1, & x > a, \\ 0, & x < a. \end{cases}$$

2. Find the Fourier transform of the function $f(x) = \begin{cases} 1 - x^2, & |x| \leq 1, \\ 0, & |x| > 1, \end{cases}$ and hence evaluate $\int_0^\infty \left(\frac{x\cos x - \sin x}{x^3}\right)\cos\left(\frac{x}{2}\right)dx$.
3. Prove that Fourier transform of $e^{-\frac{x^2}{2}}$ is self-reciprocal.
4. Find the inverse Fourier transform of $F(k) = \frac{1}{\sqrt{2\pi}}\frac{a}{b}e^{-|k|(b-a)}, b > a > 0$.
5. Find the Fourier transform of $e^{-a|x|}, -\infty < x < \infty$, and hence find $F^{-1}\left\{\frac{1}{(9+k^2)(4+k^2)}\right\}$.

Practical No. 11: Fourier sine and cosine transforms

1. Find the convolution $(f * g)(x)$ if
   (a) $f(x) = \cos x,\ g(x) = e^{-a|x|},\ a > 0$.
   (b) $f(x) = \chi_{[a,b]}(x),\ g(x) = x^2$.
2. Using the Parseval formula, prove that $\int_0^\infty \frac{t^2}{(t^2+1)^2}dt = \frac{\pi}{4}$.
3. Show that Fourier sine and cosine transforms of x^{n-1} are $\frac{\Gamma(n)}{k^n}\sin\frac{n\pi}{2}$ and $\frac{\Gamma(n)}{k^n}\cos\frac{n\pi}{2}$ respectively.
4. Find the Fourier sine transforms of $f(x) = \frac{e^{-ax}}{x}$ and use it to evaluate $\int_0^\infty \tan^{-1}\left(\frac{x}{a}\right)\sin x\, dx$.
5. Find the Fourier cosine transforms of $f(x) = \frac{1}{1+x^2}$.

Practical No. 12: Finite Fourier sine and cosine transforms

1. Find the finite Fourier sine and cosine transforms of $f(x) = 2x, 0 \leq x \leq 4$.
2. Find the finite Fourier sine and cosine transforms of $f(x) = \begin{cases} x, & 0 \leq x \leq \frac{\pi}{2}, \\ \pi - x, & \frac{\pi}{2} \leq x \leq \pi. \end{cases}$
3. Find the solution of the integral equation $\int_{-\infty}^\infty f(x-\xi)f(\xi)d\xi = \frac{1}{x^2+a^2}$.
4. Solve the integral equation $\int_{-\infty}^\infty \frac{f(t)}{(x-t)^2+a^2}dt = \frac{1}{x^2+b^2}, b > a > 0$.
5. Solve the integral equation $f(t) + 4\int_{-\infty}^\infty e^{-|x-t|}f(t)dt = e^{-|x|}$.

Practical No. 13: Applications of Fourier Transforms

1. Solve: $\frac{\partial u}{\partial t} = \alpha^2\frac{\partial^2 u}{\partial x^2}, -\infty < x < \infty, t \geq 0$ with conditions $u(x,0) = f(x)$.
2. Solve: $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}, (x > 0, t > 0)$ with the boundary conditions:
   (i) $u(0,t) = 0$ when $t > 0$,
   (ii) $u(x,t) = \begin{cases} 1, & 0 < x < 1, \\ 0, & x \geq 1, \end{cases}$ when $t = 0$, and
   (iii) $u(x,t)$ is bounded.
3. Solve: $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ with the boundary conditions:
   (i) $\frac{\partial u(0,t)}{\partial x} = 0$ for $t > 0$,
   (ii) $u(x,0) = \begin{cases} x, & 0 \leq x \leq 1, \\ 0, & x > 1, \end{cases}$ and
   (iii) $u(x,t)$ is bounded for $x > 0, t > 0$.

4. Using finite Fourier transform, solve $\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2}$ with the boundary conditions: $u(0,t)=0, u(4,t)=0$, and $u(x,0)=2x$, where $0<x<4, t>0$.
5. Solve $\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2},\ 0<x<6, t>0$, with the boundary conditions: $\frac{\partial u(0,t)}{\partial x}=0, \frac{\partial u(6,t)}{\partial x}=0, u(x,0)=2x$.

Practical No. 14: Z-transforms

1. Find $Z\{f(n)\}$if $f(n)=e^{-an}\sin(bn), n\geq 0$.
2. Find $Z\{f(n)\}$if $f(n)=2^n\cos(3n+2), n\geq 0$.
3. Find $Z\{f(n)\}$if $f(n)=3^n\sinh(\alpha n), n\geq 0$.
4. Show that $\sum_{n=0}^{\infty}\frac{x^{n+1}}{n+1}=\log(1+x)$.
5. Find the sum of the series $\sum_{n=0}^{\infty}a^n\sin(nx)$.

Practical No. 15: Inverse Z-transforms

1. Find the inverse Z-transform of $F(z)=\frac{3z^2-z}{(z-1)(z-2)^2}$.
2. Find $Z^{-1}\left[\frac{10z}{(z-2)(z-1)}\right]$ by using inversion integral method.
3. Use the convolution theorem to find the inverse of $\frac{z^2}{(z-3)(z-2)}$.
4. Use the Convolution theorem to show that $Z^{-1}\left\{\frac{z(z+1)}{(z-1)^3}\right\}=n^2$.
5. Solve: $u_{n+1}=u_n+u_{n-1},\ u_1=u_0=1$.

Recommended Books:

- Burton David M.(2007), *Elementary Number Theory*, (Sixth Edition), Tata McGraw-Hill Edition, New Delhi.
- Debnath Lokenath, Bhatta Dambaru (2007), Integral Transforms and Their Applications (Second Edition), Taylor and Francis Group, LLC.
- Bracewell Ronald N. (2000), The Fourier Transform and Its Applications (Third Edition), McGraw-Hill

Reference Books:

- Apostol T. M.(1972), *Introduction to Analytic Number Theory*,(student Edition), Springer International.
- Hari Kishan (2019), *Number Theory*, Krishna Prakashan Media (p) Ltd, Meerat
- Davies Brian (2002), Integral Transforms and Their Applications (Third Edition), Springer-Verlag, New York.

Method of Teaching:

- Classroom teaching, Lecture method, ICT based teaching

T.Y. B.Sc. (Mathematics): Semester-VI
Discipline Specific Core (DSC) Course
MTH-361: Measure and integration theory

Total Hours: 45 **Credits: 3**

Course objectives:

- To know scope and importance of measure theory and its applications.
- To explain the concept of Measurable Sets and Measurable functions with their properties.
- To discuss Lebesgue integral for bounded functions, Some fundamental theorems and metric space $L^2[a, b]$.

Course outcomes:

After successful completion of this course, a student is expected to:

- Learn measurable sets. Learn the concept of Sets of measure zero.
- Understand why a more sophisticated theory of integration and measure is needed.
- Show that certain functions are measurable.
- Understand properties of the Lebesgue integrals.

Unit-I: Measurable Sets **(11 h)**

1.1 Length of open and closed sets
1.2 Inner and outer measure of a set
1.3 Measurable sets and Properties of measurable sets
1.4 Symmetric difference of two measurable sets
1.5 Cantor's ternary sets

Unit-II: Measurable functions **(11 h)**

2.1 Real valued measurable functions
2.2 Sequence of measurable functions
2.3 Supremum and infimum of measurable functions
2.4 Almost everywhere concept

Unit-III: Lebesgue integral for bounded functions **(12 h)**

3.1 Measurable partition, Refinement, Lower and Upper Lebesgue sum and Lebesgue integrals
3.2 Existence of Lebesgue integral for bounded function.
3.3 Properties of Lebesgue integral for bounded measurable functions
3.4 Lebesgue integral for bounded function over a set of finite measure
3.5 Non-negative valued function
3.6 Positive and negative part of a function
3.7 Definition and properties of $\int_E f$ where f is nonnegative valued function in $L[a, b]$

Unit-IV: Some fundamental theorems and metric space $L^2[a, b]$ **(11 h)**

4.1 Lebesgue dominated convergence theorem
4.2 Fatou's Lemma
4.3 Square integrable function
4.4 Schwartz inequality, Minkowski's inequality

Recommended Book:

- Goldberg R.R.(1976), *Methods of Real Analysis*, Oxford & IBH Publishing Co. PVT. LTD, 2nd Edition. (Chapter-11: 11.1,11.2,11.3, 11.4,11.5,11.6,11.7, 11.8, 11.9)

Reference Books:

- Barra G. D. (2003), *Measure Theory and Integration*, Woodhead Publishing; 2 Ed.
- Jain P. K., Gupta V. P. (2019), *Lebesgue Measure and integration,* New Age International Publishers; Third edition.

Method of Teaching:

- Classroom teaching, Lecture method, ICT based teaching

T.Y. B.Sc. (Mathematics): Semester-VI
Discipline Specific Core (DSC) Course
MTH-362: Metric spaces

Total Hours: 45 **Credits: 3**

Course objectives:

- Introduction of metric as a generalization of distance function and basic concepts in metric spaces.
- To explain the concept of sequence and complete metric space with their properties.
- To discuss compactness, and sequential compact spaces and their properties along with continuity.

Course outcomes:

After successful completion of this course, a student is expected to:

- Understand the Euclidean distance function on $\mathbb{R}^n$ and appreciate its properties, and state and use the Triangle and Reverse Triangle Inequalities for the Euclidean distance function on $\mathbb{R}^n$.
- Explain the definition of continuity for functions from $\mathbb{R}^m$ to $\mathbb{R}^n$ and determine whether a given function from $\mathbb{R}^m$ to $\mathbb{R}^n$ is continuous.
- Explain the geometric meaning of each of the metric space properties (M1) –(M3) and be able to verify whether a given distance function is a metric.
- Distinguish between open and closed balls in a metric space and be able to determine them for given metric spaces.
- Define convergence for sequences in a metric space and determine whether a given sequence in a metric space converges.
- State the definition of continuity of a function between two metric spaces.

Unit-I: Metric Spaces **(11 h)**

1.1 Equivalence and Countability
1.2 Metric Spaces
1.3 Limits in Metric Spaces

Unit-II: Continuous functions on Metric Spaces **(11 h)**

2.1 Reformulation of definition of continuity in Metric Spaces.
2.2 Continuous function on Metric Spaces.
2.3 Open Sets
2.4 Closed Sets
2.5 Homeomorphisms.

Unit-III: Connected Metric Spaces and Complete of Metric Spaces **(12 h)**

3.1 More about Sets

3.2 Connected Set
3.3 Bounded and Totally bounded sets
3.4 Complete Metric Spaces
3.5 Properties of Complete Metric Spaces
3.6 Contraction Mapping on Metric Spaces.

Unit-IV: Compactness of Metric Spaces **(11 h)**

4.1 Compact Metric Spaces.
4.2 Continuous function on compact Metric Spaces.
4.3 Continuity of inverse function
4.4 Uniform Continuity

Recommended Book:

- Goldberg R.R. (1976), *Methods of Real Analysis*, Oxford & IBH Publishing Co. PVT. LTD, 2nd Edition,. Chapter I : 1.5 ,1.6 , Chapter IV : 4.2, 4.3, Chapter V : 5.2,5.3, 5.4, 5.5 Chapter VI : 6.1,6.2,6.3.6.4,6.5,6.6,6.7,6.8

Reference Books:

- Malik S. C., Arora Savita (2010), *Mathematical Analysis*, Second Edition, New Age International Pvt. Ltd., New Delhi.
- Somsundaram D. And Chaudhari B. (2018), *A First Course in Mathematical Analysis*, Narosa Publishing House, New Delhi.

Method of Teaching:

- Classroom teaching, Lecture method, ICT based teaching

T.Y. B.Sc. (Mathematics): Semester-VI
Discipline Specific Core (DSC) Course
MTH-363: Linear algebra

Total Hours: 45 **Credits: 3**

Course objectives:

- To know scope and importance of elementary Linear algebra and its applications.
- To study problems in many branches of Mathematics such as Algebra, Engineering Mathematics , Cryptography etc.
- To know properties of Linearly dependent and independent sets, basis, eigen values and eigen vectors.

Course outcomes:

After successful completion of this course, a student is expected to:

- Understand subspaces, linear span and their properties which is one of the building blocks of pure mathematics.
- Explain basis and dimensions, Extension theorem.
- Explain concepts of Linear transformations, kernel and image of linear transformation.
- Learn eigen values and eigen vectors, Diagonalisation of Matrices.

Unit-I: Vector Spaces **(11 h)**

1.1 Vector spaces, Subspaces, Examples
1.2 Necessary and sufficient conditions for a subspace
1.3 Addition, Intersection and union of subspaces
1.4 Quotient space
1.5 Linear span and properties

Unit-II: Basis and Dimensions **(12 h)**

2.1 Linear dependence and independence
2.2 Basis and dimension of finite dimensional vector spaces
2.3 Extension theorem
2.4 Theorems on basis and dimensions

Unit-III: Linear Transformations **(11 h)**

3.1 Linear transformation and examples
3.2 Kernel and image of linear transformations
3.3 Isomorphism theorems
3.4 Rank-nullity theorem
3.5 Algebra of linear transformations
3.6 Singular and non-singular linear transformations
3.7 Invertible linear transformations

Fundhi

Unit-IV: Eigen values and Eigen vectors **(11 h)**

4.1 Matrix polynomial, Characteristics polynomial

4.2 Eigen values and Eigen vectors

4.3 Cayley-Hamilton theorem

4.4 Minimum polynomial

4.5 Matrix representation of linear operator and linear transformation

4.6 Diagonalization of Matrices

Recommended Books:

- Gopalkrishnan N. S. (2015), *University Algebra*, New Age Int. Pvt. Ltd.
- Lipschutz S. (1987), *Theory and Problems of Linear Algebra,* Schaum's outline series, SI(Metric) edition, McGraw Hill Book Company.
- Dubey Balaram (2007), *Introductory Linear Algebra,* Asian books Int. Pvt. Ltd.

Reference Books:

- Khanna V. K., Bhambri S. K. (2013), *Course in Abstract Algebra*, Vikas Publishing House Pvt. Ltd. New Delhi.
- Vasishtha A. R. and Sharma J. N. (2014), *Linear Algebra*, Krishna Publication, Meerut.

Method of Teaching:

- Classroom teaching, Lecture method, ICT based teaching

T.Y. B.Sc. (Mathematics): Semester-VI
Discipline Specific Core (DSC) Course
MTH-364: Partial differential equations

Total Hours: 45 **Credits: 3**

Course objectives:

- To know the concept of a partial differential equation.
- To study Charpit's Method, Jacobi's Method and Monge's Method.
- To know the application of partial differential equation.

Course Outcomes:

After successful completion of this course, a student is expected to:

- Understand the order and degree of partial differential equation .
- Solve a partial differential equations using Charpit's and Jacobi's methods.
- Extract information from partial derivative models in order to interpret reality.

Unit-I: Origin of Partial Differential Equations **(11 h)**

1.1 Definition of partial differential equation, order and degree of partial differential Equation
1.2 Linear and non-linear partial differential equation
1.3 Classification of first order partial differential equations
1.4 Origin of partial differential equations
1.5 Derivation of partial differential equation by elimination of arbitrary constants and arbitrary functions

Unit-II: Linear and Non-linear Partial Differential Equations of Order One **(12 h)**

2.1 Lagrange's equations and Lagrange's method of solving $Pp + Qq = R$
2.2 Integral surface passing through a given curve
2.3 Surfaces orthogonal to a given system of surfaces
2.4 Complete integral (or complete solution), particular integral, singular integral (or singular solution) and general integral (or general solution)
2.5 Geometrical interpretation of integrals of $f(x, y, z, p, q) = 0$

Unit-III: Compatible Systems **(10 h)**

3.1 Method of getting singular integral directly from the partial differential equation of first order
3.2 Compatible system of first-order equations
3.3 Condition for system of two first order partial differential equation to be compatible
3.4 A particular case of Compatible System

Unit-IV: General Methods of Solving Partial Differential Equations **(12 h)**

4.1 Charpit's method and examples
4.2 Special type (a) $f(p, q) = 0$, (b)$f(z, p, q) = 0$, (c) $g(x, p) = f(y, q)$

4.3 Examples on type (a), (b) and (c).
4.4 Jacobi's method and examples
4.5 The origin second order equations
4.6 Non-linear equation of second order
4.7 Monge's method of solving $Rr + Ss + Tt = V$

Recommended Book:

- Raisinghania M. D. (2017), *Ordinary and Partial Differential Equations*, S. Chand and Company Pvt Ltd, Nineteenth Edition.
  (Part-III: 1.1-1.12, 2.1-2.2, 2.14-2.17, 3.1-3.10, 3.14-3.22, 9.1-9.4)

Reference Books:

- Amaranath T.(2003), *An Elementary Course in Partial Differential Equations*, Alpha Science International Ltd, Second Edition.
- Sneddon Ian N. (2006), *Elements of Partial Differential Equations*, McGraw-Hill, Dover Edition.

Method of Teaching:

- Classroom teaching, Lecture method, ICT based teaching

T.Y. B.Sc. (Mathematics): Semester-VI
Discipline Specific Core (DSC) Course
MTH-365: Optimization techniques

Total Hours: 45 **Credits: 3**

Course objectives:

- To study a linear programming problem (LPP).
- To study the simplex method to solve linear programming problem.
- To study the simplex method for unbounded, alternative and infeasible solutions of LPP.
- To study the initial basic feasible solution of transportation problem (TP).
- To study the two person zero sum game, saddle point, maximin-minimax principal.
- To study 2×2 games without saddle point.
- To study graphical method to solve m×2 and 2×n games.
- To study dominance property.

Course outcomes:

After successful completion of this course, a student is expected to:

- Solve the linear programming problem by graphical method and simplex method.
- Learn the unbounded, alternative and infeasible solutions of LPP by graphical and simplex method.
- Understand the standard and canonical form of LPP.
- Find the optimal solution of TP by MODI method.
- Solve the solution of assignment problems by Hungerian Method.
- Understand the unbalanced, balanced, maximization, restricted AP and alternative solution of AP.
- Understand the two person zero sum game, saddle point, maximin-minimax principal.
- Use of dominance property to find the solution games.

Unit-I: Linear Programming Problem (LPP) **(12 h)**

1.1 Formation of LPP
1.2 Solution of LPP by graphical method
1.3 Standard and Canonical forms of LPP
1.4 Simplex Algorithm
1.5 Solution of LPP by simplex method
1.6 Artificial variable technique (Big M method)
1.7 Special cases in LPP:
(a) Unbounded solution, (b) Alternate solution, and (c) Infeasible solution

Unit-II: Transportation Problem (TP) **(11 h)**

2.1 General Transportation Problem
2.2 Transportation Table. Methods for finding IBFS:

a) North –West corner rule b) Matrix minima method (Least cost method)
c) Vogel's approximation method (VAM)

2.3 Optimality test and optimization of solution to TP by U-V method (MODI). Special cases in TP:

(a) Alternate solution (b) Maximization TP (c) Unbalanced TP
(d) Restricted TP (e) Degeneracy in TP

Unit-III: Assignment Problem (AP) **(11 h)**

3.1 Mathematical Formulation of Assignment problem
3.2 Hungerian method for solving AP
3.3 Special cases in AP:

(a) Alternate solution (b) Maximization AP
(c) Unbalanced AP (d) Restricted AP

Unit-IV: Game Theory **(11 h)**

4.1 Two person-zero sum games
4.2 Pure and mixed strategies, value of a game
4.3 Maxmin and Minimax principles and saddle point
4.4 Solution of 2×2 game by algebraic method and oddment method
4.5 Game without saddle points-mixed strategies Graphical solution of m×2 and 2×n games
4.6 Dominance Property

Recommended Book:

- Swarup Kanti, Gupta P. K., Man Mohan (2004), *Operations Research,* S. Chand and Sons, Educational Publishers, New Delhi. Twelfth Edition. (Chapter No.- 3, 4, 10, 11)

Reference Books:

- Sharma S. D., Ramnath K. (2012), *Operations Research,* Meerut Publication.
- Gupta Prem Kumar (2014), *Operations Research*, S. Chand and Company pvt Ltd. New Delhi 7th Edition.

Method of Teaching:

- Classroom teaching, Lecture method, ICT based teaching

T.Y. B.Sc. (Mathematics): Semester-VI
Discipline Specific Core (DSC) Course
MTH-366: Dynamical systems

Total Hours: 45 **Credits: 3**

Course objectives:

- To introduce the main features of dynamical systems, particularly as they arise from systems of ordinary differential equations as models in applied mathematics.
- To model physical phenomena whose state (or instantaneous description) changes over time.
- To know the basic mathematical skills for the qualitative solving of low dimensional systems of ordinary differential equations in continuous time, phase portraits and bifurcations.

Course outcomes:

After successful completion of this course, a student is expected to:

- Confidently analyze a small system of ordinary differential equations and produce a quantitatively accurate local map and a qualitatively accurate global phase portrait.
- Understand how the mathematical changes from bifurcations change the structure of the phase portrait.
- Describe dynamical systems geometrically and represent them graphically via phase plane analysis.

Unit-I: First-Order Equations **(11 h)**

1.1 System of first order differential equations
1.2 The Logistic Population Model
1.3 Constant Harvesting and Bifurcations,
1.4 Periodic Harvesting and Periodic Solutions
1.5 Computing the Poincaré Map
1.6 Exploration: A Two-Parameter Family

Unit-II: Planar Linear Systems **(11 h)**

2.1 Conversion of second order differential equations into a system of differential equations
2.2 Planar Linear Systems
2.3 Solving Linear Systems
2.4 The Linearity Principle

Unit-III: Phase Portraits for Planar Systems **(11 h)**

3.1 Real distinct eigen values
3.2 Concept of saddle, source and sink types of equilibrium points
3.3 Complex eigen values
3.4 Repeated eigen values

Ftandhi

3.5 Changing coordinates

Unit-IV: Classification of Planar Systems and Exponential of Matrix **(12 h)**

4.1 The Trace-determinant plane

4.2 Dynamical classification

4.3 Exponential of operators

4.4 Solving a system of first order differential equations by using the exponential of a matrix.

Recommended Books:

- Morris Hirsch, S. Smale and Devaney, (2004), *Differential Equations*, Dynamical Systems and Introduction to Chaos, Academic Press, Elsevier, Second Edition. (Chapter-1: 1.1-1.6, Chapter-2: 2.1-2.7, Chapter-3: 3.1-3.4, Chapter-4: 4.1-4.2, Chapter-6: 6.4)
- Lawrence Perko, (2001), *Differential Equations and Dynamical Systems*, Springer-Verlag, New York, Third Edition. (Chapter-1: 1.1-1.4)

Reference Book:

- Morris Hirsch and Stephen Smale, *Differential Equations, Dynamical System and Linear Algebra*, Academic Press, Elsevier, First Edition, 1974.

Method of Teaching:

- Classroom teaching, Lecture method, ICT based teaching

T.Y. B.Sc. (Mathematics): Semester-VI
Skill Enhancement Course (SEC)
MTH-360: Vector analysis

Total Hours: 30 **Credits: 2**

Course objective:

- To know the concept of vectors algebra and vector valued functions
- To study operators like del and curl and line and surface integrals.
- Compute the curl and the divergence of vector fields.

Course Outcomes:

After successful completion of this course, a student is expected to:

- Understand scalar and vector products
- Understand vector valued functions and their limits and continuity and use them to estimate velocity and acceleration of partials.
- Calculate the curl and divergence of a vector field.
- Set up and evaluate line integrals of functions along curves.

Unit-I: The Dot and Cross Product **(8 h)**

1.1 Scalar Product
1.2 Vector Product
1.3 Scalar Triple Product
1.4 Vector Product of Three Vectors
1.5 Reciprocal Vector

Unit-II: Vector functions **(8 h)**

2.1 Vector functions of a single variable.
2.2 Limits and continuity.
2.3 Differentiability, Algebra of differentiation.
2.4 Curves in space, Velocity and acceleration.
2.5 Vector function of two or three variables.
2.6 Limits, Continuity, Partial Differentiation

Unit-III: Gradient, Divergence and Curl **(7 h)**

3.1 The vector differentiation operator ∇
3.2 Gradient, Divergence and curl
3.3 Formulae involving ∇

Unit-IV: Vector Integration **(7 h)**

4.1 Ordinary integrals of vectors
4.2 Line integrals
4.3 Surface integrals

Recommended Book:

- Spiegel Murray R, (1959) *Vector Analysis*, Schaum's Series, McGraw-Hill, First Edition. (Chapter-2, Chapter-3,Chapter-4, Chapter-5)

Reference Book:

- Shanti Narayan, Mittal P. K., (2017) *Vector Calculus*, S. Chand & Co., New Delhi, Revised Edition.

Method of Teaching:

- Classroom teaching, Lecture method, ICT based teaching

T.Y. B.Sc. (Mathematics): Semester-VI
Discipline Specific Core (DSC) Course
MTH-367: Practical course based on MTH-361 and MTH-362

Total Hours: 60 **Credits: 2**

Course objectives:

- To know problem solving skills in Theory of measure and integration.
- To know problem solving skills in Lebesgue integral.
- To get hands on training for solving problems of Metric space.

Course outcomes:

After successful completion of this course, a student is expected to:

- Understand and solve problems on measurable set.
- Apply Fundamental theorems to solve problems.
- Solve problems on Metric space.

Sr. No.	Topic Particular	Hours
1	Measurable sets	4
2	Properties of Measurable sets	4
3	Measurable functions	4
4	Properties of Measurable functions	4
5	Lebesgue Integral for bounded functions	4
6	Lebesgue Integral for unbounded functions	4
7	Some Fundamental Theorems	4
8	Metric spaces	4
9	Limits in metric spaces	4
10	Open Sets in Metric space	4
11	Closed sets and Homeomorphisms	4
12	Connected Metric spaces	4
13	Complete metric spaces	4
14	Compact metric spaces	4
15	Continuous functions on compact metric spaces	4

Practical No. 1: Measurable sets

1. Let $E \subset [a,b]$ then show that $\underline{m}E \leq \overline{m}E$. If $E \subset [a,b]$ and $\overline{m}E = 0$ Prove that E is measurable and $mE = 0$.
2. Show that $E \subset [a,b]$ is measurable if and only if given $\epsilon > 0$ there exist a closed set $F \subset E$ and open set $G \supset E$ such that $|G| - |F| < \epsilon$
3. Show that singleton set is measurable and its measure is zero .
4. Find the measure of the following set $E = \bigcup_{k=1}^{\infty} \left\{x: \frac{1}{2^k} < x < \frac{1}{2^{k-1}}\right\}$.
5. If E_1 is a measurable subset of $[a,b]$ and if $mE_2 = 0$ prove that $E_1 \cup E_2$ is measurable

Practical No. 2: Properties of Measurable sets

1. If E_1 and E_2 are subset of $[a,b]$ then show that $\overline{m}E_1 + \overline{m}E_2 \geq \overline{m}(E_1 \cup E_2) + \overline{m}(E_1 \cap E_2)$.
2. If $E_1, E_2, \ldots$ are any subset of $[a,b]$ then show that $\overline{m}(\cup E_n) \leq \sum_{n=1}^{\infty} \overline{m}E_n$.
3. If E_1 and E_2 are measurable subset of $[a,b]$ then show that $E_1 - E_2$ is also measurable . In addition if $E_2 \subset E_1$, then show that $m(E_1 - E_2) = mE_1 - mE_2$.
4. If E_1 and E_2 are subset of $[a,b]$, if the symmetric difference of E_1 and E_2 has measure zero, and if E_1 is measurable, then E_2 is measurable.
5. If $E \subset [a,b]$ then show that $\overline{m}E + \underline{m}E' = b - a$.

Practical No. 3: Measurable functions

1. If $f(x) = \frac{1}{x}, 0 < x < 1\ ,\ f(0) = 5, f(1) = 7$, then prove that f is measurable on $[0,1]$.
2. Show that the subset E of $[a,b]$ is measurable if and only if its characteristic function Ψ_E is measurable .
3. Let f and g be functions on $[a,b]$. If $f(x) = g(x)$ almost everywhere* $(a \leq x \leq b)$ and if f is measurable, then show that g is measurable .
4. Show that every continuous function g on $[a,b]$ is measurable .
5. Show that the function f defined on $\mathbb{R}$ by $f(x) = \begin{cases} x+5 & , x < -1 \\ 2 & , -1 \leq x \leq 0 \\ x^2 & , x > 0 \end{cases}$ is measurable.

Practical No. 4: Properties of Measurable functions

1. If f is measurable function on $[a,b]$, and if $c \in \mathbb{R}$, then show that the functions $f + c$ and cf are measurable .
2. If f and g are measurable functions on $[a,b]$, then show that $f + g$ and $f - g$ are measurable function .
3. If f and g are measurable functions on $[a,b]$, then show that $f\,g$ and $\frac{f}{g}$ are measurable function .
4. If f is measurable function on $[a,b]$, then show that the following conditions hold .
   a. For every $s \in R$ the set $\{\ x \mid f(x) \geq s\ \}$ is a measurable set .
   b. For every $s \in R$ the set $\{\ x \mid f(x) < s\ \}$ is a measurable set .
   c. For every $s \in R$ the set $\{\ x \mid f(x) \leq s\ \}$ is a measurable set .
5. If f is measurable function on $[a,b]$, then show that the inverse image under f of any interval (bounded, unbounded, closed, open, half open etc .) is a measurable set.

Practical No. 5: Lebesgue Integral for bounded functions

1. Let $f(x) = \begin{cases} 2 & , 0 \leq x < 1, \\ 4 & , 1 \leq x < 2, \\ 3 & , 2 \leq x < 3, \\ 2 & , 3 \leq x \leq 4. \end{cases}$
   a) If σ is the subdivision $\{ 0,1,2,3,4 \}$ of $[0,4]$, calculate $U[f;\sigma]$.
   b) For $k = 2,3,4$, let E_k be the inverse image under f of $[k, k+1[$. Show that $P = \{E_2, E_3, E_4\}$ is a measurable partition of $[0,4]$.
   c) Calculate $U[f;P]$ and $L[f;P]$.
2. If $f \in \mathcal{L}[a,b]$, if $E \subset [a,b]$ and $= 0$, show that $\int_E f = 0$.

3. If E_1 and E_2 are measurable subsets of $[a, b]$, and If $f \in \mathcal{L}[a, b]$, Prove that $\int_{E_1} f + \int_{E_2} f = \int_{E_1 \cup E_2} f + \int_{E_1 \cap E_2} f$.
4. If E is measurable subset of $[a, b]$, then show that $\int_E 1 = mE$.
5. If f is bounded function in $\mathcal{L}[a, b]$, and if g is a bounded function on $[a, b]$ such that $f(x) = g(x)$ almost everywhere $(a \leq x \leq b)$, then show that $g \in \mathcal{L}[a, b]$ and $\int_a^b g = \int_a^b f$.

Practical No. 6: Lebesgue Integral for unbounded functions

1. Let $f(x) = log\frac{1}{x}, 0 < x \leq 1$. Find $2f$.
2. If $f(x) = \frac{1}{x^p}, 0 < x \leq 1$, then prove that $f \in \mathcal{L}[0,1]$ if $p < 1$ and that $\int_0^1 f = \frac{1}{1+p}$.
3. Evaluate $\mathcal{L} \int_0^1 f$, where $f(x) = \frac{1}{\sqrt[3]{x}}, 0 < x \leq 1$.
4. Show that f is not Lebesgue integrable, where $f(x) = \begin{cases} \frac{1}{x} & ,0 < x \leq 1, \\ 19 & ,x = 0 \end{cases}$.
5. Find f^+ and f^-, where $f(x) = \frac{1}{2} + sinx, 0 \leq x < 2\pi$.

Practical No. 7: Some Fundamental Theorems

1. If $f \in \mathcal{L}[a, b]$ then show that for given $\epsilon > 0$ there exist $\delta > 0$ such that $\left|\int_E f\right| < \epsilon$, whenever E is measurable subset of $[a, b]$ with $mE < \delta$.
2. For $n \in I$, let $f_n(x) = \begin{cases} 2n, & \frac{1}{2n} \leq x \leq \frac{1}{n}, \\ 0, & x \in \left(0, \frac{1}{2n}\right) \cup \left(\frac{1}{n}, 1\right). \end{cases}$

   Calculate $\int_0^1 [\lim_{n\to\infty} f_n(x)dx]$ and $\lim_{n\to\infty} \int_0^1 f_n(x)dx$.
3. Using Lebesgue dominated convergence theorem, Evaluate $\lim_{n\to\infty} \int_0^1 f_n(x)dx$, where $f_n = \frac{n^{3/2} \cdot x^{3/2}}{1+n^2x^2}$, $0 \leq x < 1$.
4. Let f and g be measurable functions on $[a, b]$. If f and g are square integrable, then show that $fg \in \mathcal{L}[a, b]$ and $\left|\int_a^b fg\right| \leq \left(\int_a^b f^2\right)^{1/2} \left(\int_a^b g^2\right)^{1/2}$.
5. Let f and g be measurable functions on $[a, b]$. If f and g are square integrable, then show that $f + g$ is also square integrable and

$$\left[\int_a^b (f + g)^2\right]^{1/2} \leq \left(\int_a^b f^2\right)^{1/2} + \left(\int_a^b g^2\right)^{1/2}$$

Practical No. 8: Metric spaces

1. For $P(x_1, y_1), Q(x_2, y_2) \in \mathbb{R}^2$, define $\sigma(P, Q) = |x_1 - x_2| + |y_1 - y_2|$. Show that σ is a metric.
2. For $P(x_1, y_1), Q(x_2, y_2) \in \mathbb{R}^2$, define $\tau(P, Q) = \max(|x_1 - x_2|, |y_1 - y_2|)$. Show that τ is a metric.
3. Let ℓ^∞ denote the set of all bounded sequences of real numbers. If $x = \{x_n\}_{n=1}^\infty$ and $y = \{y_n\}_{n=1}^\infty$ are points in ℓ^∞, define $\rho(x, y) = l.u.b. |x_n - y_n|$. Show that ρ is a metric.
4. Define $\rho: \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ by $\rho(x, y) = |x - y|$. Show that $(\mathbb{R}, \rho)$is a metric space.

5. Define $\rho: \mathbb{R} \times \mathbb{R} \to [0, \infty)$ by $(x, y) = \begin{cases} 0, & x = y \\ 1, & x \neq y \end{cases}$. Show that $(\mathbb{R}, \rho)$is a metric space.

Practical No. 9: Limits in metric spaces

1. Show that a sequence of points in any metric space cannot converge to two distinct limits.
2. Show that every convergent sequence is a Cauchy sequence. Is the converse true, Justify
3. Show that every Cauchy sequence in $\mathbb{R}_d$ is convergent.
4. Show that if $\{x_n\}_{n=1}^{\infty}$ is a convergent sequence in $\mathbb{R}_d$ then there exists $N \in$ I such that $x_n = x_{n+1} = x_{n+2} = \cdots$.
5. If $\{x_n\}$ is a Cauchy sequence of points in a metric space M, and if $\{x_n\}$ has a subsequence which converges to $x \in M$. Prove that $\{x_n\}$ itself convergent to x.

Practical No. 10: Open Sets in Metric spaces

1. If A and B are open subsets of $\mathbb{R}^1$ prove that $A \times B$ is an open subset of $\mathbb{R}^2$
2. Let f and g be continuous real-valued functions on the metric space M. Let A be the set of all $x \in M$ such that $f(x) < g(x)$. prove that A is open.
3. Give an example of subsets A and B of $\mathbb{R}^2$ such that all three of the following conditions hold.
   (a) Neither A nor B is open $(b) A \cap B = \emptyset$ $(c) A \cup B$is open.
4. Which of the following subsets of $\mathbb{R}^2$ are open?
   $(a)\ \{(x, y)/x + y = 1\}$ $(b)\{(x, y)/x + y > 1\}$ $(c)\{(x, y)/x$ and y rational$\}$.
5. If G_1 and G_2 are open subsets of the metric space M, then show that $G_1 \cap G_2$ is also open.

Practical No. 11: Closed sets and Homeomorphisms

1. If A and B are open subsets of $\mathbb{R}^1$ prove that $A \times B$ is a closed subset of $\mathbb{R}^2$
2. Let f be continuous real- valued function on the metric space M Let A be the set of all $x \in M$ such that $f(x) \geq 0$. Prove that A is closed.
3. Show that $\mathbb{R}^1$ and $\mathbb{R}_d$ are not homeomorphic.
4. Let M be a metric space and let $A \subset B \subset M$. If A is dense in B and if B is dense in M, prove that A is dense in M.
5. Prove that $(0, \infty)$ (with absolute value metric) is homeomorphic with $\mathbb{R}^1$.

Practical No. 12: Connected Metric spaces

1. True or false? If A and C are connected subsets of the metric space M, and if $A \subset B \subset C$, then B is connected.
2. If A is connected subsets of the metric space M, prove that $\bar{A}$ is connected.
3. If A is connected subsets of the metric space M, and if $A \subset B \subset \bar{A}$, prove that B is connected.
4. Let B be a subset of the metric space M. Prove that B is dense in M if and only if B is $\epsilon -$ dense in M for every $\epsilon > 0$.
5. Prove that every finite subset of a metric space M is totally bounded.

Practical No. 13: Complete metric spaces

1. Prove that $\mathbb{R}_d$ is complete.
2. Prove that the interval $(0,1)$ with absolute value metric is not a complete metric space.
3. Prove that $\mathbb{R}^2$ is complete .

4. If $T(x) = x^2 \left(0 \le x \le \frac{1}{3}\right)$, Prove that T is a contraction on $\left[0, \frac{1}{3}\right]$.
5. If $T: [0,1] \to [0,1]$ and if there is a real number α with $0 \le \alpha \le 1$ such that $|T'(x)| \le \alpha \ (0 \le x \le 1)$, Where T' is derivative of T, prove that T is a contraction on $[0,1]$.

Practical No. 14: Compact metric spaces

1. Prove that every finite subset of any metric space is compact .
2. Prove that a subset A of $\mathbb{R}^2$ is compact if and only if A is closed and bounded .
3. If A and B are compact subsets of $\mathbb{R}^1$, prove that $A \times B$ is compact subset of $\mathbb{R}^2$
4. For each x in $(0,1)$ let I_x denote the open interval $\left(\frac{x}{2}, \frac{x+1}{2}\right)$. Show that the family $\mathcal{g}$ of all such I_x is an open covering of $(0,1)$ which admits no finite subcovering of $(0,1)$
5. Let A be a subset of the metric space (M, ρ) . If (A, ρ) is compact, Then show that A is a closed subset of (M, ρ).

Practical No. 15: Continuous functions on compact metric spaces

1. Let $f: \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = \sin x, \ \forall x \in \mathbb{R}$. Examine whether $f(x)$ is uniformly continuous or not.
2. Show that $f(x) = x^2, \forall x \in [0,1]$ is uniformly continuous on $[0,1]$
3. Let f be continuous function from the compact metric space M_1 into the metric space M_2. Show that the range $f(M_1)$ of f is also compact.
4. Give an example of: a) compact, Complete and connected metric space
   b) compact, Complete but not connected metric space
5. Prove that connected subset of $\mathbb{R}_d$ is compact.

Recommended Book:

- Goldberg R.R. (1976), *Methods of Real Analysis*, Oxford & IBH Publishing Co. PVT. LTD, 2nd Edition.

Reference Books:

- Barra G. D. (2003), *Measure Theory and Integration*, Woodhead Publishing; 2nd Edition.
- Jain P. K., Gupta V. P. (2019), *Lebesgue Measure and integration,* New Age International Publishers; Third edition.
- Malik S. C., Arora Savita (2010) , *Mathematical Analysis*, Second Edition, New Age International Pvt. Ltd., New Delhi.
- Somsundaram D., Chaudhari B. (2018), *A First Course in Mathematical Analysis*, Narosa Publishing House, New Delhi.

Method of Teaching:

- Classroom teaching, Lecture method, ICT based teaching

T.Y. B.Sc. (Mathematics): Semester-VI
Discipline Specific Core (DSC) Course
MTH-368: Practical course based on MTH-363 and MTH-364

Credits: 2

Total Hours: 60

Course objectives:

- To know problem solving skills in theory of basis, dimensions and partial differential equations.
- To know problem solving skills in linear transformations and compatible system.
- To know problem solving skills in theory of eigen values and eigen vectors, Cayley Hamilton theorem, diagonalisation of matrices, Jacobi's method and Monge's method.
- To know the different methods of solving a partial differential equation.

Course outcomes:

After successful completion of this course, a student is expected to:

- Understand and solve problems on linearly dependent, linearly independent sets and integral surface.
- Apply isomorphism theorems to solve problems.
- Explain concepts and solve problems on bases, dimensions, Cayley Hamilton theorem and compatible system.
- Apply concepts of Eigen values and Eigen vectors to solve problems in matrix theory.
- Solve a partial differential equation by using Charpit's method, Monge's method, Jacobi's method.
- Solve problems on compatible system.

Sr. No.	Topic Particular	Hours
1	Vector spaces and Subspaces	4
2	Quotient space and Linear span	4
3	Linearly dependent, Linearly independent sets and basis	4
4	Dimension of vector spaces	4
5	Linear Transformations	4
6	Invertible linear transformations	4
7	Eigen values and Eigen vectors	4
8	Cayley-Hamilton theorem and Diagonalisation of Matrices	4
9	Origin of partial differential equations-I	4
10	Origin of partial differential equations-II	4
11	Integral surface-I	4
12	Integral surface-II	4
13	Compatible systems	4
14	Complete Integrals	4
15	Jacobi's and Monge's Method	4

Practical No. 1: Vector spaces and Subspaces

1. Let V be an abelian group of positive real numbers under multiplication. Define scalar

multiplication in V by $ax = x^a, a \in \mathbb{R},\ x \in V$. Show that V is a $\mathbb{R}$-vector space.

2. Let $V = F^n$ be a F-vector space and $W = \{(a_1, a_2, \cdots, a_n) \in V : a_1 = 0\}$. Show that W is a subspace of V.
3. Let $V = \mathbb{R}^n$ and $W = \{(a_1, a_2, \cdots, a_n) \in V : a_1 + a_2 + \cdots + a_n = 3\}$. Is W a subspace of V? Justify.
4. Let $V = \mathbb{R}^3$, be a $\mathbb{R}$-vector space and $W_1 = \{(a, b, 0) \in V : a, b \in \mathbb{R}\}$, $W_2 = \{(0, 0, c) \in V : c \in \mathbb{R}\}$ be subspaces of V. Is $W_1 \cup W_2$ a subspace of V? Justify.
5. Let $V = \mathbb{R}^3$, be a $\mathbb{R}$-vector space and $W = \{(x, y, z) \mid x, y, z \in \mathbb{R}\ and\ x - 3y + 4z = 0\}$. Show that W is a subspace of V.

Practical No. 2: Quotient space and Linear span

1. Let W be a subspace of a F-vector space V and $x, y \in V$. Show that $x + W = y + W \Leftrightarrow x - y \in W$.
2. Write the vector $v = (1, -2, 5)$ as linear combination of the vectors $e_1 = (1, 1, 1)$, $e_2 = (1, 2, 3)$, $e_3 = (2, -1, 1)$.
3. Find the value of k such that the vector $u = (1, -2, k)$ in $\mathbb{R}^3$ is a linear combination of the vectors $v = (3, 0, -2)$ and $w = (2, -1, -5)$.
4. Show that the vectors $u = (1, 2, 3)$, $v = (0, 1, 2)$ and $w = (0, 0, 1)$ generates $\mathbb{R}^3$.
5. Let $V = \mathbb{R}^2$, $W_1 = \{(a, 0) \in V : a \in \mathbb{R}\}\ and\ W_2 = \{(0, b) \in V : b \in \mathbb{R}\}$. Show that $V = W_1 \oplus W_2$.

Practical No. 3: Linearly dependent, linearly independent sets and basis

1. Let V be a F-vector space. If x, y, z are linearly independent vectors in V, then prove that $x + y,\ y + z,\ z + x$ are also linearly independent.
2. Let V be a F-vector space. If x, y, z are linearly independent vectors in V, then prove that $x + y,\ y - yz,\ x - 2y + z$ are also linearly independent.
3. Let V be a F-vector space and $S \subseteq V$. If there are two elements x, y in S such that $x = \alpha y$ for some $\alpha \in F$, then show that S is a linearly dependent set.
4. Let V be a F-vector space with a basis $\{v_1,\ v_2, \cdots, v_n\}$ and $a \in F, a \neq 0$. Show that $\{av_1, av_2,\ \cdots, av_n\}$ is a basis of V.
5. Let $V = F_n[x]$ be the vector space of all polynomials of degree $\leq n$ over a field F. Show that $\{1, x, x^2, \cdots, x^n\}$ is a basis of V.

Practical No. 4: Dimension of vector spaces

1. Prove that the set $S = \{(1, 2, 1), (-1, 1, 0), (5, -1, 2)\}$ is a basis of $\mathbb{R}^3$ over $\mathbb{R}$.
2. Let $W = \langle x - 1, x^2 + x - 1, x^2 - x + 1\rangle$ be a subspace of a $\mathbb{R}$-vector space $\mathbb{R}_2[x]$. Find $dim\left(\frac{\mathbb{R}_2[x]}{W}\right)$.
3. Let W_1, W_2 be distinct subspaces of a F-vector space V. If $dim(V) = n, dim(W_1) = n - 1 = dim(W_2)$. Find $dim(W_1 \cap W_2)$.
4. Does there exist subspaces W_1, W_2 of a F-vector space V such that $dim(V) = 7$, $dim(W_1) = 4 = dim(W_2)$ and $dim(W_1 \cap W_2) = 0$?
5. Let W_1, W_2 be subspaces of a F-vector space V. If $dim(V) = 8, dim(W_1) = 6$, $dim(W_2) = 5$, then find possible dimensions of $W_1 \cap W_2$.

Practical No. 5: Linear Transformations

1. Let $T: \mathbb{R}^2 \to \mathbb{C}$ be a map defined by $T(a, b) = a + ib$, for all $(a, b) \in \mathbb{R}^2$. Show that T is a linear transformation.
2. Let F be a field & $V = F^{n+1}, V' = F_n[x]$ = the set of all polynomials of degree $\leq n$ over F. Show that $T: V \to V'$ defined by $T(a_0, a_1, a_2, \cdots, a_n) = a_0 + a_1x + a_2x^2 + \cdots + a_nx^n$ is a linear transformation.
3. Let $T: \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation defined by $T(x + y) = (x, x + y, y)$. Find the range and rank of T.
4. Let $T: \mathbb{R}^4 \to \mathbb{R}^3$ be a linear transformation defined by $T(x_1, x_2, x_3, x_4) = (x_1 - x_4, x_2 + x_3, x_3 - x_4)$. Find the kernel and nullity of T.
5. Let V be a F-vector space and $\dim(V) = n$. If $Range(T) = Ker(T)$, then show that n is even.

Practical No. 6: Invertible linear transformations

1. Show that the linear operator on $\mathbb{R}^3$ defined by $T(a, b, c) = (a + b + c,\ b + c,\ c)$ is non singular and find its inverse.
2. Let $T: \mathbb{R}^3 \to \mathbb{R}^3$ be the linear transformation defined by $T(x, y, z) = (3x,\ x - y,\ 2x + y + z)$. Prove that T is invertible and find the formula for T^{-1}.
3. Find $T(a, b, c)$ where $T: \mathbb{R}^3 \to \mathbb{R}$ is a linear transformation defined by $T(1, 1, 1) = 3,\ T(0, 1, -2) = 1,\ T(0, 0, 1) = -2$.
4. Find a linear transformation $T: \mathbb{R}^3 \to \mathbb{R}^3$ whose image is generated by $(1, 2, 3)$ and $(4, 5, 6)$.
5. Let $T : V \to V$ be a linear transformation such that T is not onto. Show that there exists $v \in V - \{0\}$ such that $T(v) = 0$.

Practical No. 7: Eigen values and Eigen vectors

1. Find the eigen values and corresponding eigen vectors of the matrix $A = \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix}$.
2. Find the characteristics roots, their corresponding vectors and the basis for the eigen space of the matrix $A = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 1 & -1 \\ 0 & 2 & 4 \end{bmatrix}$.
3. Show that $-1, -1,\ 3$ are the eigen values of the matrix $A = \begin{bmatrix} -9 & 4 & 4 \\ -8 & 3 & 4 \\ -16 & 8 & 7 \end{bmatrix}$. Find the eigen space corresponding to eigen value $\lambda = 3$.
4. Find the eigen values of $A = \begin{bmatrix} 2 & -2 & 3 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{bmatrix}$. Find the eigen space corresponding to eigen value $\lambda = 1$.
5. Let A be a non-singular square matrix of order n. If λ is a characteristic root of A then show that $\frac{|A|}{\lambda}$ is a characteristic root of $adj(A)$.

Practical No. 8: Cayley-Hamilton theorem and Diagonalisation of Matrices

1. Find the characteristic equation of $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ & verify that it is satisfied

by A & hence find A^{-1}.

2. Verify Cayley-Hamilton theorem for the matrix $A = \begin{bmatrix} 1 & 2 & 0 \\ 2 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$ and hence obtain A^{-1}.
3. Find minimum polynomial of $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$.
4. Show that $A = \begin{bmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{bmatrix}$ is a diagonalizable matrix.
5. Show that $A = \begin{bmatrix} -3 & 1 & -3 \\ -7 & 5 & -1 \\ -6 & 6 & -2 \end{bmatrix}$ is not a diagonalizable matrix.

Practical No. 9: Origin of partial differential equations-I

1. For each of the following partial differential equations, classify as linear or non-linear and find its order and degree:
   a) $\frac{\partial^2 z}{\partial x^2} = (1 + \frac{\partial z}{\partial y})^{\frac{1}{2}}$
   b) $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = xyz$
   c) $\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = z + xy$
2. Find the partial differential equation of all planes which are at a constant distance 'a' from the origin
3. Find the partial differential equation of all right circular cones whose axes coincide with z-axis
4. From differential equation by eliminating constant A and P from $z = Ae^{pt} sinpx$
5. Find a partial differential equation by eliminating a and b from $z = ax + by + a^2 + b^2$.

Practical No. 10: Origin of partial differential equations-II

1. Eliminate a and b from $z = axe^y + \left(\frac{1}{2}\right) a^2 e^{2y} + b$.
2. Find the differential equation of all sphere of radius λ having centre in the xy-plane.
3. From a partial differential equation by eliminating the arbitory function f from the equation $x + y + z = f(x^2 + y^2 + z^2)$.
4. From a partial differential equation by eliminating the arbitory function f from the equation $z = e^{ax+b} f(x^2 + y^2 + z^2)$.
5. Find a partial differential equation by eliminating a, b, c from $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$.

Practical No. 11: Integral surface-I

1. Find the general solution of $xp + yq = z$.
2. Find the general solution of $x^2p + y^2q = (x + y)\,z$.
3. Find the integral surface of the linear partial differential equation $x(y^2 + z)p - y(x^2 + z)q = (x^2 - y^2)z$ which contains the straight line $x + y = 0, z = 1$.
4. Find the integral surface of the linear partial differential equation $2y(z - 3)p + (2x - z)q = y(2x - 3)$ which pass through the circle $z = 0, x^2 + y^2 = 2x$.
5. Find the general integral of the partial differential equation $(2xy - 1)p + (z - 2x^2)q = 2(x - yz)$ and also particular integral which passes through the line $x = 1, y = 0$.

Practical No. 12: Integral surface-II

1. Write down the system of equation for obtaining the general equation of surface orthogonal to family given by $x(x^2 + y^2 + z^2) = c_1y^2$.
2. Find the surface which is orthogonal to the one parameter system $y = cxy(x^2 + y^2)$ which passes through the hyperbola $x^2 - y^2 = a^2, z = 0$.
3. Find the family of surface orthogonal to the family of surfaces given by the differential equation $(y + z)p + (z + x)q = x + y$.
4. Find the family orthogonal to $\varphi[z(x + y)^2,\ x^2 - y^2] = 0$.
5. Solve: $\frac{y^2z}{x}p + xzq = y^2$.

Practical No. 13: Compatible systems

1. Show that equation $xp - yq = x$ and $x^2p + q = xz$ are compatible and find their solution.
2. Show that the equation $z = px + qy$ is compatible with any equation $f(x, y, z, p, q) = 0$ which is homogeneous in x, y, z.
3. Show that $\frac{\partial z}{\partial x} = 7x + 8y - 1$ and $\frac{\partial z}{\partial y} = 9x + 11y - 2$ are not compatible.
4. Show that $p = 6x + 3y$ and $q = 3x - 4y$ are compatible and hence solve.
5. Show that $p = ax + hy + g$ and $q = hx + by + f$ are compatible and hence solve.

Practical No. 14: Complete Integrals

1. Find a complete, singular and general integral of $(p^2 + q^2)y = qz$.
2. Find a complete and singular integral of $2xz - px^2 - 2qxy + pq = 0$.
3. Find the general integral and the singular integral of $2(z + xp + yq) = yp^2$.
4. Find the complete integral of $px + qy = z(1 + pq)^{\frac{1}{2}}$.
5. Solve completely simultaneous equation $z = px + qy$ and $2xy(p^2 + q^2) = z(yp + xq)$.

Practical No. 15: Jacobi's and Monge's Method

1. Find the complete integral of $9(p^2z + q^2) = 4$.
2. Find a complete integral by Jacobi's method of the equation $2x^2y\left(\frac{\partial u}{\partial x}\right)^2\left(\frac{\partial u}{\partial z}\right) =$

$x^2 \frac{\partial u}{\partial y} + 2y \left(\frac{\partial u}{\partial x}\right)^2$.

3. Solve: $p^2 x + q^2 y = z$ by Jacobi's method.
4. Show that a complete integral of the equation $f\left(\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial u}{\partial z}\right) = 0$ is $u = ax + by + \theta(a, b)z + c$, where a, b and c are arbitrary constants and $f(a, b, \theta) = 0$.
5. Solve: $x^2 r - y^2 t = xp - yq$ by Monge's method.

Recommended Books:

- Gopalkrishnan N. S. (2015), *University Algebra*, New Age Int. Pvt. Ltd.
- Lipschutz S. (1987), *Theory and Problems of Linear Algebra,* Schaum's outline series, SI(Metric) edition, McGraw Hill Book Company,.
- Dubey Balaram (2007), *Introductory Linear Algebra,* Asian books Int. Pvt. Ltd.
- Raisinghania M. D.(2017), *Ordinary and Partial Differential Equations*, S. Chand and Company Pvt Ltd, Nineteenth Edition.

Reference Books:

- Khanna V. K., Bhambri S. K. (2013), *Course in Abstract Algebra*, Vikas Publishing House Pvt. Ltd. New Delhi.
- Vasishtha A. R., Sharma J. N. (2014), *Linear Algebra*, Krishna Publication, Meerut.
- Amaranath T. (2003), *An Elementary Course in Partial Differential Equations*, Alpha Science International Ltd, Second Edition.
- Sneddon Ian N. (2006), *Elements of Partial Differential Equations*, McGraw-Hill, Dover Edition.

Method of Teaching:

- Classroom teaching, Lecture method, ICT based teaching

T.Y. B.Sc. (Mathematics): Semester-VI
Discipline Specific Core (DSC) Course
MTH-369: Practical course based on MTH-365 and MTH-366

Total Hours: 60 **Credits: 2**

Course objectives:

- To develop analytical and computational skills.
- To get hands on training in solving problems of Optimization techniques and Dynamical systems.
- To know the method of solving the LPP
- To know the different methods of solving the transportation problem
- To know the concept of assignment problem and game theory.
- To understand fundamental concepts related to modeling time dependent phenomena.
- To think analytically when creating a mathematical model from a description of the process and analyze it using mathematical and numerical methods, both manually and using technology.
- To know the qualitative analysis of a dynamical system using many different analytic tools.

Course outcomes:

After successful completion of this course, a student is expected to:

- Develop problem solving analytical and computational skills.
- Solve problems on LPP by Graphical method and simplex method
- Solve problems of transportation problems and assignment problems
- Solve problems of game theory
- Describe qualitatively the behaviour of the solution of a dynamical system without necessarily finding the exact solution.
- Draw phase portraits and interpret them in several applications from biology, physics, chemistry and engineering.
- Identify gradient fields and use their special properties to obtain phase portrait.

Sr. No.	Topic Particular	Hours
1	Linear Programming Problem (LPP)	4
2	Simplex Method	4
3	Transportation Problem (TP)-I	4
4	Transportation Problem (TP)-II	4
5	Assignment Problem (AP)-I	4
6	Assignment Problem (AP)-II	4
7	Game Theory	4
8	First-Order Equations	4
9	Bifurcation Diagram	4
10	Planar Linear Systems-I	4
11	Planar Linear Systems-II	4
12	Phase Portraits for Planar Systems-I	4
13	Phase Portraits for Planar Systems-II	4
14	Classification of Planar Systems and Exponential of Matrix	4
15	Exponential of Matrix	4

Practical No. 1: Linear Programming Problem (LPP)

1. Use graphical method to solve the LPP: Min $Z = x_1 + 0.5x_2$
   subject to the constraints $3x_1 + 2x_2 \leq 12$
   $$5x_1 \leq 10$$
   $$x_1 + x_2 \geq 8$$
   $$-x_1 + x_2 \geq 4$$
   $$x_1, x_2 \geq 0$$
2. Use graphical method to solve the LPP: Max $Z = 2x_1 + 4x_2$
   subject to the constraints $x_1 + 2x_2 \leq 5$
   $$x_1 + x_2 \leq 4$$
   $$x_1, x_2 \geq 0$$
   Is this LPP has alternative solution? If yes, find it.
3. Using graphical method, show that the following LPP has unbounded solution.
   Max $Z = 6x_1 + x_2$
   subject to the constraints $2x_1 + x_2 \geq 3$
   $$x_2 - x_1 \geq 0$$
   $$x_1, x_2 \geq 0$$
4. Using graphical method show that the following LPP has infeasible solution.
   Max $Z = x_1 + x_2$
   subject to the constraints $x_1 + x_2 \leq 1$
   $$-3x_1 + x_2 \geq 3$$
   $$x_1, x_2 \geq 0$$
5. Reduce the following LPP to its standard form:
   Max $Z = x_1 + x_2 + 4x_3$
   subject to the constraints $-2x_1 + 4x_2 \leq 4$
   $$x_1 + 2x_2 + x_3 \geq 5$$
   $$2x_1 + 3x_2 \leq 2$$
   $$x_1, x_2, x_3 \geq 0$$

Practical No. 2: Simplex Method

1. Use simplex method to solve the LPP: Max $Z = 4x_1 + 10x_2$
   subject to the constraints $2x_1 + x_2 \leq 50$
   $$2x_1 + 5x_2 \leq 100$$
   $$2x_1 + 3x_2 \leq 90$$
   $$x_1 \geq 0, x_2 \geq 0$$
2. Using Big-M method show that the following LPP does not possess any feasible solution. Max $Z = 3x_1 + 2x_2$
   subject to the constraints $2x_1 + x_2 \leq 2$
   $$3x_1 + 4x_2 \geq 12$$
   $$x_1 \geq 0, x_2 \geq 0$$
3. Using Big-M method show that the following LPP has alternative solution.
   Max $Z = 6x_1 + 4x_2$
   subject to the constraints $2x_1 + 3x_2 \leq 30$

$$3x_1 + 2x_2 \leq 24$$
$$x_1 + x_2 \geq 3$$
$$x_1 \geq 0, x_2 \geq 0$$

4. Using simplex method solve the LPP: Max $Z = 3x_1 + 4x_2$
   subject to the constraints $x_1 + x_2 \leq 4$
   $$2x_1 + x_2 \leq 5$$
   $$x_1 \geq 0, x_2 \geq 0$$
5. Use simplex method to solve the LPP: Max $Z = 3x_1 + 2x_2$
   subject to the constraints $x_1 + x_2 \leq 4$
   $$x_1 - x_2 \leq 2$$
   $$x_1 \geq 0, x_2 \geq 0$$

Practical No. 3: Transportation Problem (TP)-I

1. Obtain IBFS of TP by using North-West Corner rule

	D	E	F	G	Availability
A	11	13	17	14	250
B	16	18	14	10	300
C	21	24	13	10	400
Requirements	200	225	275	250	

2. Obtain IBFS of TP by using Matrix Minima Method

	D_1	D_2	D_3	D_4	Capacity
O_1	1	2	3	4	6
O_2	4	3	2	0	8
O_3	0	2	2	1	10
Demand	4	6	8	6	

3. Obtain IBFS of TP by using Vogel's Approximation Method

	D	E	F	G	Availability
A	11	13	17	14	250
B	16	18	14	10	300
C	21	24	13	10	400
Demand	200	225	275	250	

4. Convert the following unbalanced TP into balanced TP.

		Destinations					
		I	II	III	IV	V	Supply
Sources	A	4	3	26	38	30	160
	B	3	2	34	34	198	280
	C	3	3	24	28	30	240
	Demand	1	1	200	120	240	

5. Obtain IBFS by VAM and solve the transportation problem for minimum cost.

	D_1	D_2	D_3	Supply
S_1	2	7	4	5
S_2	3	3	1	8
S_3	5	4	7	7
S_4	1	6	2	14
Demand	7	9	18	

Practical No. 4: Transportation Problem (TP)-II

1. Solve the following transportation problem for maximum profit

	I	II	III	IV	Supply
A	40	25	22	33	100
B	44	35	30	30	30
C	38	38	28	30	70
Demand	40	20	60	30	

Obtain an optimal basic feasible solution of the following degeneracy problem

	W_1	W_2	W_3	Available
F_1	7	3	4	2
F_2	2	1	3	3
F_3	3	4	6	5
Demand	4	1	5	

2. Solve the following transportation problem

	A	B	C	D	E	Supply
P	4	1	3	4	4	60
Q	2	3	2	2	3	35
R	3	5	2	4	4	40
Demand	22	45	20	18	30	

3. Obtain the initial basic feasible solution of the following example by north-west corner method as well as by VAM method.

	I	II	III	IV	Supply
A	7	3	5	5	34
B	5	5	7	6	15
C	8	6	6	5	12
D	6	1	6	4	19
Demand	21	25	17	17	

4. Find initial basic feasible solution for following transportation problem by using least cost method

	D_1	D_2	D_3	D_4	Supply
O_1	3	4	6	3	30
O_2	3	5	7	10	50
O_3	2	6	5	7	70
Demand	22	41	44	43	

Practical No. 5: Assignment Problem (AP)-I

1. Solve following AP.

	I	II	III	IV
A	2	3	4	5
B	4	5	6	7
C	7	8	9	8
D	3	5	8	4

Is there exist alternative solution? If Yes, Find it.

2. A departmental head has four subordinates and four tasks to be performed. The subordinates differs in efficiency and the tasks differ in their intrinsic difficulty. His estimate, of the time each man would take to perform each task, is given in the matrix below:

Tasks \ Men	E	F	G	H
A	18	26	17	11
B	13	28	14	26
C	38	19	18	15
D	19	26	24	10

How should the tasks be allocated, one to a man, so as to minimize total manhours?

3. Solve the following assignment problem for maximum profit.

	1	2	3	4
A	16	10	14	11
B	14	11	15	15
C	15	15	13	12
D	13	12	14	15

4. The following is the cost matrix of assigning 4 clerks to 4 key punching jobs. Find the optimal assignment if clerk I cannot be assigned to job 1:

Clerk \ Job	I	II	III	IV
A	--	5	2	0
B	4	7	5	6
C	5	8	4	3
D	3	6	6	2

What is the minimum total cost?

5. Convert the following unbalanced AP into balanced AP and solve it for minimization.

	A	B	C
W	9	26	15
X	13	27	6
Y	35	20	15
Z	18	30	20

Practical No. 6: Assignment Problem (AP)-II

1. Solve the following assignment problem for minimum cost

	I	II	III	IV	V
A	1	3	2	3	6
B	2	4	3	1	5
C	5	6	3	4	6
D	3	1	4	2	2
E	1	5	6	5	4

2. A company has four territories and four salesmen available for the assignment. The cells(rupees in lacks) for each salesmen to be assign each territory is given below

salesmen \ Territories	I	II	III	IV
A	84	70	56	42
B	60	50	40	30
C	60	50	40	30
D	48	40	32	24

Find the optimum assignment schedule that maximizes total profit.

3. The owner of a small machine shop has four machinists available to assign for jobs for a day. Five jobs are offered with expected profit (in rupees) on each job as follows

Jobs

machinists		A	B	C	D	E
	1	62	78	50	101	82
	2	71	84	61	73	59
	3	87	92	111	71	81
	4	48	64	87	77	80

Find the assignment of machinists to the jobs that will result in a maximum profit.

4. Solve the following assignment problem to minimize the total design time

Design time project

Originers		A	B	C	D
	I	12	10	10	8
	II	14	∞	15	11
	III	6	10	16	4
	IV	8	10	9	7

5. There are five pilots and five flights given in the table

Flight No.

Pilot		I	II	III	IV	V
	A	8	2	X	5	4
	B	10	9	2	8	4
	C	5	4	9	6	X
	D	3	6	2	8	7
	E	5	6	10	4	3

Find the optimal solution of the assignment problem.

Practical No. 7: Game Theory

1. Find the best strategy of each player and the value of game.

Player B

Player A					
	9	3	1	8	0
	6	5	4	6	7
	2	4	3	3	8
	5	6	2	2	1

2. A and B play a game in which each has three coins 5p ,10p and 20p each player selects the point without the knowledge of coin, if the sum of coin is an odd amount, A wins B's coin and if the sum of coin is even then B wins A's coin. Find the best strategy for player A &B and the value of game.

3. Solve the following game by arithmetic method

Player B

Player A		
	5	-1
	0	12

4. Solve the following 2 X 4 game by graphical method.

Player B

Player A

3	3	4	0
5	4	3	7

5. Solve the following game by using dominance principle

Player B

Player A

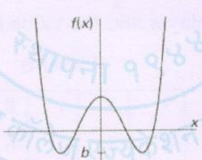
2	-2	4	1
6	1	12	3
-3	2	0	6
2	-3	7	1

Practical No. 8: First Order Equations

1. Find the general solution of the differential equation $x' = ax + 3$ where a is a parameter. What are the equilibrium points for this equation? For which values of a are the equilibria sinks? For which are they sources?
2. For each of the following differential equations, find all equilibrium solutions and determine if they are sinks, sources, or neither.
   (a) $x' = x^3 - 3x$ (b) $x' = \cos x$
3. Sketch the phase line of the differential equation $x' = x^4 - x^2$.
4. Sketch the phase line of the differential equation $x' = \sin^2 x$.
5. Sketch the phase line of the differential equation $x' = |1 - x^2|$.

Practical No. 9: Bifurcation Diagram

1. Sketch the bifurcation diagram of $x' = x^2 - ax$, where a is a parameter.
2. Sketch the bifurcation diagram of $x' = x^3 - x + b$, where b is a parameter.
3. Sketch the bifurcation diagram of $x' = x(1-x) - h$, where h is a parameter.
4. Consider the function $f(x)$ whose graph is displayed in the following figure



   (a) Sketch the phase line corresponding to the differential equation $x' = f(x)$.
   (b) Let $g_a(x) = f(x) + a$. Sketch the bifurcation diagram corresponding to the family of differential equations $x' = g_a(x)$.
5. Consider a first-order linear equation of the form $x = ax + f(t)$ where $a \in \mathbb{R}$. Let $y(t)$ be any solution of this equation. Prove that the general solution is $y(t) + ce^{at}$ where $c \in \mathbb{R}$ is arbitrary.

Practical No. 10: Planar Linear Systems-I

1. Find the eigenvalues and eigenvectors of each of the following 2×2 matrices:
   (a) $\begin{pmatrix} 3 & 1 \\ 1 & 3 \end{pmatrix}$ and (b) $\begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}$

2. Find the general solution of $X' = \begin{pmatrix} 1 & 2 \\ 0 & 3 \end{pmatrix} X$.
3. Find the general solution of $X' = \begin{pmatrix} 1 & 2 \\ 3 & -3 \end{pmatrix} X$.
4. Sketch the direction field for the system $x' = y,\ y' = -x$.
5. Sketch the direction field for the system $X' = \begin{pmatrix} 1 & 2 \\ 3 & 6 \end{pmatrix} X$.

Practical No. 11: Planar Linear Systems-II

1. Sketch the direction field for the system $X' = \begin{pmatrix} 1 & 2 \\ 1 & 0 \end{pmatrix} X$.
2. Prove that two vectors $V = (v_1, v_2)$ and $W = (w_1, w_2)$ are linearly independent if and only if $det \begin{pmatrix} v_1 & w_1 \\ v_2 & w_2 \end{pmatrix} \neq 0$.
3. Describe all possible 2×2 matrices whose eigenvalues are 0 and 1.
4. Consider the 2×2 matrix $A = \begin{pmatrix} a & 1 \\ 0 & 1 \end{pmatrix}$. Find the value a_0 of the parameter a for which A has repeated real eigenvalues.
5. Find the general solution of the second order differential equation $x'' + 3x' + 2x = 0$.

Practical No. 12: Phase Portraits for Planar Systems-I

1. Find the general solution of the harmonic oscillator equation $x'' + x' + x = 0$.
2. Sketch the phase portrait of $X' = \begin{pmatrix} -1 & 0 \\ 0 & 2 \end{pmatrix} X$.
3. Sketch the phase portrait of $X' = \begin{pmatrix} -1 & -1 \\ 1 & -1 \end{pmatrix} X$.
4. Sketch the phase portrait of $X' = \begin{pmatrix} 1 & -2 \\ 2 & 0 \end{pmatrix} X$.
5. Sketch the phase portrait of $X' = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} X$.

Practical No. 13: Phase Portraits for Planar Systems-II

1. Consider the system $X' = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} X$. Find the matrix T that puts A in canonical form.
2. Consider the system $X' = \begin{pmatrix} 1 & 1 \\ -1 & 3 \end{pmatrix} X$. Find the matrix T that puts A in canonical form.
3. Consider the system $X' = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} X$.
   a) Find the matrix T that puts A in canonical form.
   b) Find the general solution of both $X' = AX$ and $Y' = (T^{-1}AT)Y$.
4. Consider the system $X' = \begin{pmatrix} 1 & 1 \\ -1 & -3 \end{pmatrix} X$.
   a) Find the matrix T that puts A in canonical form.
   b) Find the general solution of both $X' = AX$ and $Y' = (T^{-1}AT)Y$.
5. Sketch the phase portrait of $X' = AX$ where $A = \begin{pmatrix} a & 1 \\ 2a & 2 \end{pmatrix}$. For which values of a do you find a bifurcation?

Practical No. 14: Classification of Planar Systems and Exponential of Matrix

1. Prove that $H(x,y) = (x,-y)$ provides a conjugacy between $X' = \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} X$ and $Y' = \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} Y$.
2. Consider the systems $X' = \begin{pmatrix} -1 & 1 \\ 0 & 2 \end{pmatrix} X$ and $Y' = \begin{pmatrix} 1 & 0 \\ 1 & -2 \end{pmatrix} Y$. Find an explicit conjugacy between their flows.
3. Consider the systems $X' = \begin{pmatrix} 0 & 1 \\ -4 & 0 \end{pmatrix} X$ and $Y' = \begin{pmatrix} 0 & 2 \\ -2 & 0 \end{pmatrix} Y$. Find an explicit conjugacy between their flows.
4. Compute the exponentials of the following matrices: (a) $\begin{pmatrix} 2 & 0 \\ 0 & -3 \end{pmatrix}$, (b) $\begin{pmatrix} 1 & 2 \\ 0 & -1 \end{pmatrix}$, and (c) $\begin{pmatrix} 1 & 0 \\ 5 & 1 \end{pmatrix}$
5. Find the eigen values of e^A where $A = \begin{pmatrix} 2 & -1 \\ 1 & 2 \end{pmatrix}$.

Practical No. 15: Exponential of Matrix

1. Compute the exponentials of $A = \begin{bmatrix} 2 & 0 & 0 \\ 1 & 2 & 0 \\ 0 & 1 & 2 \end{bmatrix}$.
2. Find e^{At} and solve the linear system $X' = \begin{pmatrix} 3 & 1 \\ 1 & 3 \end{pmatrix} X$.
3. Find e^{At} and solve the linear system $X' = \begin{pmatrix} 1 & 3 \\ 3 & 1 \end{pmatrix} X$.
4. Show that if X is an eigenvector of A corresponding to the eigenvalue λ, then X is also an eigenvector of e^A corresponding to the eigenvalue e^{λ}.
5. Find e^{At} where $A = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 2 & 0 \\ 1 & 0 & -1 \end{pmatrix}$.

Recommended Book:

- Swarup Kanti, Gupta P. K., Man Mohan (2004), *Operations Research,* S. Chand and Sons, Educational Publishers, New Delhi. Twelfth Edition.
- Morris Hirsch, Stephen Smale and Devaney, Differential Equations, Dynamical Systems and Introduction to Chaos, Academic Press, Elsevier, Second Edition, 2004.
- Lawrence Perko, (2001), Differential Equations and Dynamical Systems, Springer-Verlag, New York, Third Edition

Reference Books:

- Sharma S. D., Ramnath K. (2012), *Operations Research,* Meerut Publication.
- Gupta Prem Kumar (2014), *Operations Research*, S. Chand and Company pvt Ltd. New Delhi 7th Edition.
- Morris Hirsch and Stephen Smale, (1974), Differential Equations, Dynamical System and Linear Algebra, Academic Press, Elsevier, First Edition.

Method of Teaching:

- Classroom teaching, Lecture method, ICT based teaching

Skills acquired and Job prospects for the Mathematics students

Skills acquired:

1. Critical thinking.
2. Problem solving.
3. Analytical thinking.
4. Quantitative reasoning.
5. Ability to manipulate precise and intricate ideas.
6. Construct logical arguments and expose illogical arguments.
7. Time management

Job prospects

Between one third and one half of all jobs requiring graduates are open to students of any discipline. Of course, mathematicians are eligible for these jobs. In addition, there are careers for which a degree in mathematics is either essential or a strong advantage. These fall into a number of general areas:

1. Scientific research, design and development.
2. Management services and computing.
3. Financial work.
4. Statistical work.
5. Teaching.
6. Postgraduate study.
7. IT Sector

Finally, a degree in mathematics does not train you for a specific job. Rather, it gives you a range of skills which enable you to enter any of a wide range of careers. It is therefore a versatile qualification. By taking a mathematics degree, you are able to make your career choice when you are 21 rather than when you are 18. Your aspirations may well have changed during the intervening years. Moreover, you will have a clearer understanding of the work you would be doing and you will have been able to talk with representatives of the companies who will wish to employ you. Three years at a university/College will broaden your horizons in many ways. There is no need to narrow your career horizon while you are still at school unless you so wish.